

Sample Problems for ECE 572 - 23.03.2012

(From Agrawal Ch. 2, pp. 72-73) (1)

1) A multimode step index fibre has a

diameter $50 \mu\text{m}$ and intermodal dispersion

of $\Delta T/L = 10 \text{ ns/km}$. Find NA and B

of this fibre. Take $L = 10 \text{ km}$, $n_2 = 1.45$

Solution:
$$\frac{\Delta T}{L} = \frac{NA^2}{2nc} \quad (\text{ns/km})$$

$$NA = \left(\frac{\Delta T}{L} \times 2nc \right)^{1/2} \quad \frac{\Delta T}{L} = 10 \times 10^{-9} \text{ sec/km}$$

$$= \left(10 \times 10^{-9} \times 2 \times 1.45 \times 3 \times 10^5 \right)^{1/2}$$

$$= 0.0933$$

$$B = \frac{2nc}{L NA^2} = \frac{2 \times 1.45 \times 3 \times 10^5}{10 \times 0.087} = 1 \text{ Mb/s}$$

(2)

2) A single mode fibre has $n_1 - n_2 = 0.005$.

Calculate the core radius if the fibre is to have a cutoff wavelength of $4\mu\text{m}$. Estimate the spot size W and fraction of power of the mode inside core if this fibre is used at $\lambda = 1.3\mu\text{m}$. Take $n_1 = 1.45$.

Solution: $V = qk (n_1^2 - n_2^2)^{1/2}$, $V_{\min} = 2.405$

$n_1 - n_2 = 0.005$ if $n_1 = 1.45$, then $n_2 = 1.445$

$$k = \frac{2\pi}{\lambda} = 2\pi \times 10^6, \quad q = \frac{V_{\min}}{2\pi \times 10^6 \times (1.45^2 - 1.445^2)^{1/2}}$$

$$q = 3.24 \mu\text{m}$$

From Eq. (2.2.45) of Agrawal

$$\frac{W}{a} \simeq 0.65 + 1.615 V^{-3/2} + 2.879 V^{-6}$$

(3)

If we change $\lambda = 1.3 \mu\text{m}$, then

keeping a the same V would change to

$$V = a k (n_1^2 - n_2^2)^{1/2} = 3.24 \times 10^{-6} \times \frac{2\pi}{1.3 \times 10^{-6}} (1.45^2 - 1.445^2)^{1/2}$$

$$= 1.884$$

$$\frac{w}{a} = 0.65 + 1.619 \times (1.884)^{-3/2} + 2.879 \times (1.884)^{-6}$$

$$w = 4.3481 \mu\text{m} \text{ (so field extends in cladding)}$$

$$T = \frac{P_{\text{core}}}{P_{\text{cladding}}} = 1 - \exp\left(-\frac{2a^2}{w^2}\right)$$

$$= 0.6706$$

Approximately % 67 percent of power is carried in core.

3) A $1.3 \mu\text{m}$ fibre cable system has (4)
 50 km length and requires at least $0.3 \mu\text{W}$
 at receiver. Fibre loss is 0.5 dB/km , fibre
 is spliced every 5 km with two connectors
 of each 1 dB loss, and splice loss is 0.2 dB .
 Determine the minimum power that must be
 launched into the fibre.

Solution: Total loss = Fibre loss $\times$ Length (L)

$$\begin{aligned}
 &+ \text{No of splices} \times 0.2 \text{ dB} + \text{Number of splices} \times 2 \times 1 \text{ dB} \\
 &= 0.5 \times 50 + 9 \times 0.2 \text{ dB} + 18 \times 1 = 44.8 \text{ dB}
 \end{aligned}$$

$$P_r(z=50 \text{ km}) = P_i(z=0) - 44.8 \text{ dB}$$

$$\begin{aligned}
 P_i &= -35.2288 + 44.8 = 9.5712 \text{ dBm} \\
 &= 9.0598 \text{ mW}
 \end{aligned}$$