Çankaya University – ECE Department – ECE 474 (MT)

Student Name :   Date : 09.04.2014
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Questions

1. **(70 Points)** The following two fibres are given

a) $a_1 = 3\ \mu$m, $n_1 = 1.5$, $n_2 = 1.49$
b) $a_2 = 3\ \mu$m, $n_1 = 1.5$, $n_2 = 1.4865$

They both have the same length of $L = 20$ km and are both are driven by the same light source of wavelength, $\lambda = 1.55\ \mu$m with the spectral width of $\sigma_\lambda = 0.5$ nm.
Firstly make sure that these fibres are single mode fibres. Then by using the graphs given in Figs. 1-4, evaluate for each fibre the amount of total dispersion comprising material and waveguide dispersions. From there find the maximum bit rate that can be offered by each fibre. Using these maximum bit rate capacities, evaluate the minimum source power, $P(z = 0)$ that has to be launched into each fibre so that an SNR = 40 dB can be maintained at the receiver side. Evaluate and plot the NAs of these fibres. For each fibre, find the amount of power propagating in the cladding region.

Solution : a) We find V and Δ for the two cases as follows

$$a)\ V = ak\left(n_1^2 - n_2^2\right)^{0.5} = 3\times10^{-6} \times \frac{2\pi}{\lambda} \times \left(1.5^2 - 1.49^2\right)^{0.5} = 2.1028$$

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} = \frac{1.5^2 - 1.49^2}{2\times1.5^2} = 0.0066$$

$$b)\ V = ak\left(n_1^2 - n_2^2\right)^{0.5} = 3\times10^{-6} \times \frac{2\pi}{\lambda} \times \left(1.5^2 - 1.4865^2\right)^{0.5} = 2.4418$$

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} = \frac{1.5^2 - 1.4865^2}{2\times1.5^2} = 0.009 \tag{1.1}$$

As seen for only a), single mode condition is satisfied, since in case b) $V > 2.405$. Below, we continue with the two fibres for the sake of comparison.

At $\lambda = 1.55\ \mu$m

$$f = \frac{c}{\lambda} = \frac{3\times10^8}{1.55\times10^{-6}} = 1.935\times10^{14}\ \text{Hz}\quad,\quad \omega = 2\pi f = 1.216\times10^{15}\ \text{rad/sec} \tag{1.2}$$

Using Fig. 2 of this exam paper or by reading it from the graphical output of the m file RefindexVcurves, we have

$$\frac{dn}{d\lambda} \simeq 6700 \simeq \frac{dn_{2g}}{d\lambda}$$

Material dispersion parameter : $D_M = \frac{1}{c}\frac{dn_{2g}}{d\lambda} = 22.3 \times 10^{-6}$ s/m^2

$$= 22.3 \times 10^{-6} \overbrace{\times 10^{12}}^{\text{for conversion into psec}} \overbrace{\times 10^{3}}^{\text{for conversion into km}^{-1}} \overbrace{\times 10^{-9}}^{\text{for conversion into nm}^{-1}} = 22.3 \text{ ps/km/nm} \qquad (1.3)$$

From Fig. 2.20 of Agrawal, we read approximately the same numeric value. Note that material dispersion parameter, D_M is the same for a) and b).

For waveguide dispersion parameter, we have

Waveguide dispersion parameter : $D_W = -\frac{2\pi\Delta}{\lambda^2}\left[\frac{n_{2g}^2}{n_2\omega}V\frac{d^2(Vb_n)}{dV^2} + \frac{dn_{2g}}{d\omega}\frac{d(Vb_n)}{dV}\right] \qquad (1.4)$

By using the part of (4.3.4) of Attenuation and dispersion in fibres_March 2013_HTE, we convert from the derivative with respect to wavelength into radial frequency as follows

$$\frac{d}{d\omega} = -\frac{\lambda^2}{2\pi c}\frac{d}{d\lambda} \qquad (1.5)$$

Then, inserting numeric values in (1.4) and using Fig. 3, we get

$$a)\ D_W = -\frac{2\pi \times 0.66 \times 10^{-2}}{\left(1.55 \times 10^{-6}\right)^2}\left[\frac{(1.463)^2}{1.49 \times 1.216 \times 10^{15}} \times \overbrace{0.18}^{\text{read from Fig.3}} - \frac{\left(1.55 \times 10^{-6}\right)^2 \times 6700}{2\pi \times 3 \times 10^8}\overbrace{0.97}^{\text{read from Fig.3}}\right]$$

$$= -3.532 \times 10^{-6} \text{ s/m}^2 = -3.532 \text{ ps/km/nm} \qquad (1.6)$$

$$b)\ D_W = -\frac{2\pi \times 0.9 \times 10^{-2}}{\left(1.55 \times 10^{-6}\right)^2}\left[\frac{(1.463)^2}{1.49 \times 1.216 \times 10^{15}} \times \overbrace{0.08}^{\text{read from Fig.3}} - \frac{\left(1.55 \times 10^{-6}\right)^2 \times 6700}{2\pi \times 3 \times 10^8}\overbrace{1}^{\text{read from Fig.3}}\right]$$

$$= -2.023 \times 10^{-6} \text{ s/m}^2 = -2.023 \text{ ps/km/nm} \qquad (1.7)$$

From Fig. 2.20 of Agrawal, we read slightly higher numeric value. Note that waveguide dispersion parameter, D_W ha no a) and b) distinction.

For total dispersion and fibre bandwidth, maximum bit rate that can be offered with $L = 20$ km and $\sigma_\lambda = 0.5$ nm

$$a) \ \delta T = \left(D_M + D_W\right) L \sigma_\lambda = D L \sigma_\lambda = 20 \times 18.768 \times 0.5 = 187.68 \ \text{ps}$$

$$B_t = \frac{1}{\delta T} = 5.3 \ \text{GHz} : \text{Total bandwidth offered over 20 km}$$

$$C_t \simeq B_t = 5.3 \ \text{Gb/s} : \text{Total capacity offered over 20 km}$$

$$b) \ \delta T = \left(D_M + D_W\right) L \sigma_\lambda = D L \sigma_\lambda = 20 \times 20.28 \times 0.5 = 202.8 \ \text{ps}$$

$$B_t = \frac{1}{\delta T} = 4.93 \ \text{GHz} : \text{Total bandwidth offered over 20 km}$$

$$C_t \simeq B_t = 4.93 \ \text{Gb/s} : \text{Total capacity offered over 20 km} \tag{1.8}$$

At $\lambda = 1.55 \ \mu\text{m}$, from Fig. 4, we read $\alpha = 0.3 \ \text{dB/km}$, then using (3.3) of Attenuation and dispersion in fibres_March 2013_HTE, we have

$$\alpha_p = \frac{\alpha}{4.343} = 0.069 \ \text{km}^{-1} \tag{1.9}$$

For SNR calculations, we benefit from (4.1.11) of Attenuation and dispersion in fibres_March 2013_HTE, hence

$$T_a \left(\text{absolute temprature}\right) = 273 + 20 = 293 \ {}^0\text{K}$$

$$S_n\left(f\right) = \frac{kT_a}{2} = \frac{1.38 \times 10^{-23} \times 293}{2} = 2.0217 \times 10^{-21} \ \text{J} : \text{Two side noise spectral density}$$

$$a) \ P_n = 2 S_n\left(f\right) B_t = 2 \times 2.0217 \times 10^{-21} \times 5.3 \times 10^9 = 2.143 \times 10^{-11} \ \text{W}$$

$$= 2.143 \times 10^{-5} \ \mu\text{W: Noise power} \rightarrow -76.69 \ \text{dBm}$$

$$P_r = \text{SNR(in dB)} + P_n(\text{in dBm}) = 40 - 76.69 = -36.69 \ \text{dBm} \rightarrow 0.214 \ \mu\text{W}$$

$$P\left(z = 0\right) = P_r \times \exp\left(\alpha_p L\right) = 0.85 \ \mu\text{W}$$

$$b) \ P_n = 2 S_n\left(f\right) B_t = 2 \times 2.0217 \times 10^{-21} \times 4.93 \times 10^9 = 1.99 \times 10^{-11} \ \text{W}$$

$$= 1.99 \times 10^{-5} \ \mu\text{W: Noise power} \rightarrow -77 \ \text{dBm}$$

$$P_r = \text{SNR(in dB)} + P_n(\text{in dBm}) = 40 - 77 = -37 \ \text{dBm} \rightarrow 0.199 \ \mu\text{W}$$

$$P\left(z = 0\right) = P_r \times \exp\left(\alpha_p L\right) = 0.79 \ \mu\text{W} \tag{1.10}$$

According to (2.4) of Notes on Fibre Propagation_Jan 2013_HTE, NA of the fibre is given by

$$a) \ \text{NA} = \left(n_1^2 - n_2^2\right)^{0.5} = \left[\left(1.5\right)^2 - \left(1.49\right)^2\right]^{0.5} = 0.1729 \ , \ \theta_{0c} = \sin^{-1}\left(\text{NA}\right) = 9.96^0$$

$$b) \ \text{NA} = \left(n_1^2 - n_2^2\right)^{0.5} = \left[\left(1.5\right)^2 - \left(1.4865\right)^2\right]^{0.5} = 0.2 \ , \ \theta_{0c} = \sin^{-1}\left(\text{NA}\right) = 11.58^0 \tag{1.11}$$

The related plot is similar to Fig. 1.1 of ECE474_MT-08042013_Solutions.

For percentage of power propagating in the core, from (4.4) of Notes on Fibre Propagation_Jan 2013_HTE, we get

$a)$ $V = 2.1028$, $\dfrac{P_{cladding}}{P_{total}} = \exp\left(-\dfrac{2}{w_s^2}\right) = \exp\left[-\dfrac{2}{\left(0.65 + 1.619V^{-1.5} + 2.879V^{-6}\right)^2}\right] = 0.2576$

$a)$ $V = 2.4418$, $\dfrac{P_{cladding}}{P_{total}} = \exp\left(-\dfrac{2}{w_s^2}\right) = \exp\left[-\dfrac{2}{\left(0.65 + 1.619V^{-1.5} + 2.879V^{-6}\right)^2}\right] = 0.1845 \quad (1.12)$

2. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer.

a) In graded index fibre, the projections of ray trajectory of meridional rays on the fibre cross section are elliptical, whereas those of skew rays are circular : False, in graded index fibres, the ray trajectory of meridional rays on the fibre cross section is a straight line, whereas that of skew rays is elliptical.

b) In graded index fibres, a ray becomes meridional if it is incident on fibre end face with $\theta_{x0} \neq \theta_{y0}$ or $x_0 \neq y_0$: False, according to (2.30) of Notes on Propagation in GI fibres_Feb 2013_HTE, it will become meridional, if $x_0\theta_{y0} = y_0\theta_{x0}$.

c) The number of modes propagation in a fibre is determined by the value of normalized frequency, V : True, according to Figs. 3.4, 3.5 and 3.6 of Notes on Fibre Propagation_Jan 2013_HTE.

d) Dispersion is measured along time axis : True, according to Fig. 2.1 of Attenuation and dispersion in fibres_March 2013_HTE.

e) In multimode fibres, intramodal dispersion exists : True, since intramoldal dispersion arises as a result of changes of refractive index with wavelegth.
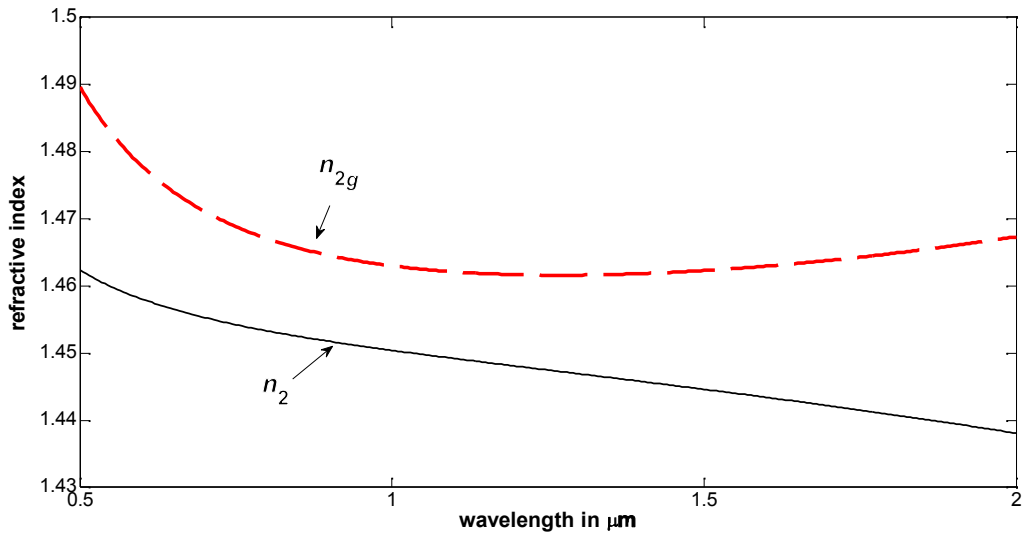
Fig. 1 Variation of cladding refractive index, n_2 and n_{2g} with wavelength.
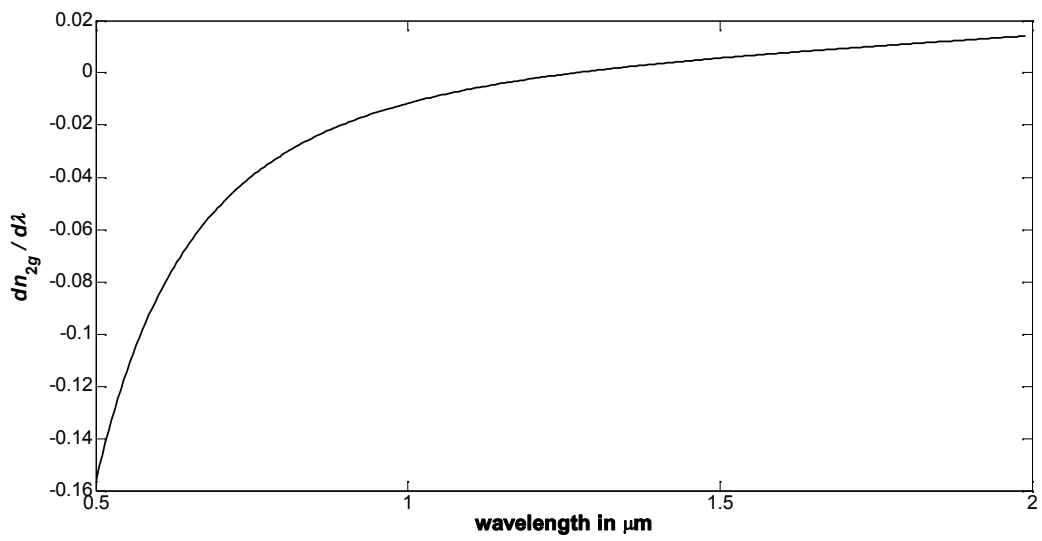

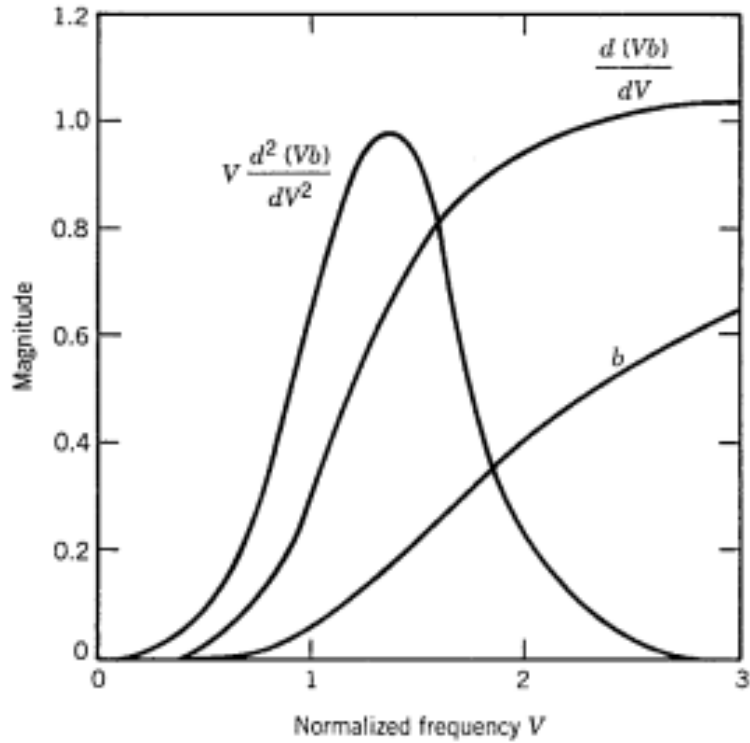Fig. 2 Variation of the derivative of n_{2g} with wavelength.

Fig. 3 Variations of b_n, $\dfrac{d\left(Vb_n\right)}{dV}$ and $\dfrac{d^2\left(Vb_n\right)}{dV^2}$ with V. Note that in our notation b is b_n (Fig. 2.9 of Agrawal).
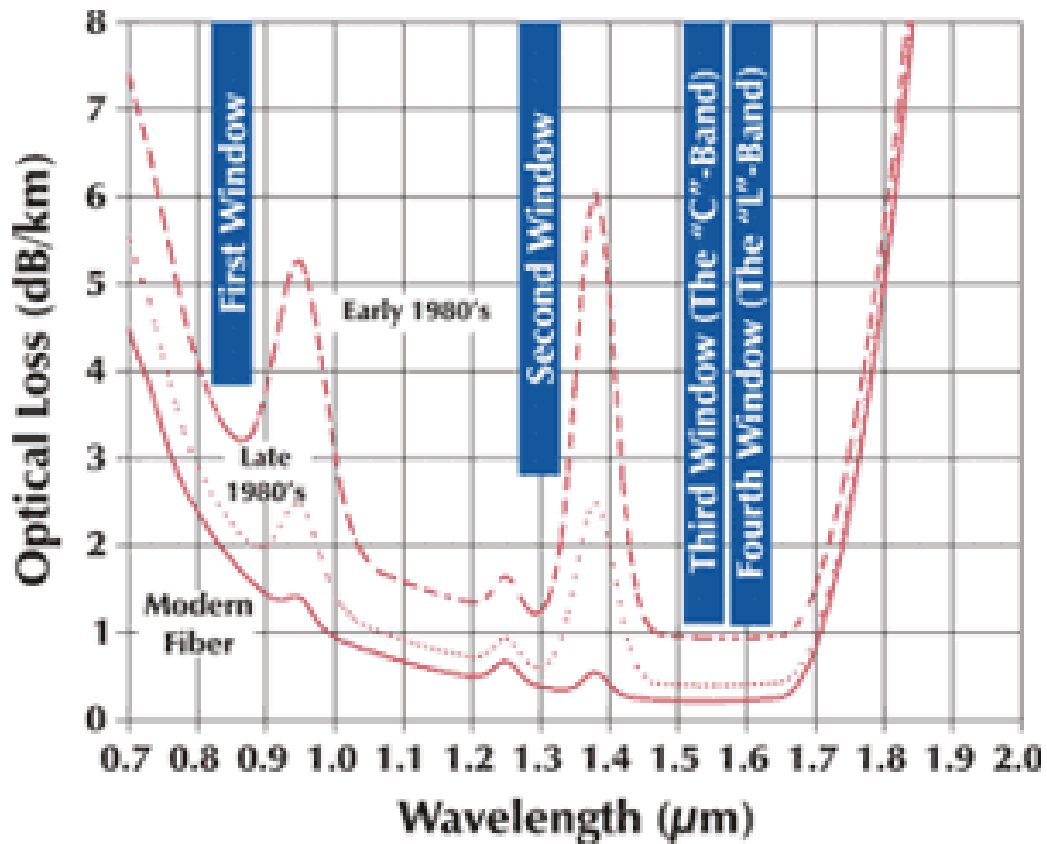

Fig. 4 Variation of fibre attenuation with wavelength.