Çankaya University – ECE Department – ECE 572

Student Name : Duration : 2 hours
Student Number : Open book exam

Questions

1. (35 Points) In a graded index fibre with a quadratic index profile, the core, cladding refractive index values and the core diameter are given as $n_1 = 1.49,\ n_2 = 1.47,\ 2a = 40\ \mu\text{m}$. With the help of Fermat principle and the associated differential equations, find for this fibre, the range and the limits of a) Propagating Meridional Rays, b) Propagating Skew Rays, c) Refracting Rays, in terms of initial launching conditions, i.e. $x_0,\ y_0,\ \theta_{x0}$ and θ_{y0}. In this respect, classify the following rays as propagating meridional, propagating skew or refracting rays

   A. $x_0 = 1.2\ \mu\text{m},\ y_0 = 1.2\ \mu\text{m},\ \theta_{x0}=0.1,\ \theta_{y0} = 0.2$
   B. $x_0 = 5\ \mu\text{m},\ y_0 = 7\ \mu\text{m},\ \theta_{x0}=0.4,\ \theta_{y0} = 0.3$
   C. $x_0 = 25\ \mu\text{m},\ y_0 = 0\ \mu\text{m},\ \theta_{x0}=0.1,\ \theta_{y0} = 0.05$

Where applicable, find for rays in A., B., and C., $r_{\min}$, $r_{\max}$ and z_p.

Solution : To classify rays according to initial launching parameters $x_0,\ y_0,\ \theta_{x0}$ and θ_{y0}, we benefit from the notes entitled, Fermat Principle_Ray Propagation inGI_kr_2012, we see that parameter z_p determines the location of $r_{\min}$, $r_{\max}$. It is clear that for propagating meridional rays, the following should be satisfied

$$r_{\min} = \left[x^2\left(z_{p\min}\right) + y^2\left(z_{p\min}\right)\right]^{0.5} = 0 \quad \text{and} \quad r_{\max} = \left[x^2\left(z_{p\max}\right) + y^2\left(z_{p\max}\right)\right]^{0.5} \le a \qquad \text{(Q1.1)}$$

The first condition states that for meridional ray should pass through the z axis, while the second condition indicates that the ray should reflect into the core before reaching the cladding. From the above mentioned notes, we get z_p as

$$z_p = \frac{a}{2\sqrt{2\Delta}}\tan^{-1}\left[\frac{2a\sqrt{2\Delta}\left(x_0\theta_{x0} + y_0\theta_{y0}\right)}{2\Delta\left(x_0^2 + y_0^2\right) - a^2\left(\theta_{x0}^2 + \theta_{y0}^2\right)}\right] = z_{p\min} \qquad \text{(Q1.2)}$$

If above z_p refers to $z_{p\min}$ as written, then $z_{p\max}$ will be

$$z_{p\max}\ \frac{a}{2\sqrt{2\Delta}}\tan^{-1}\left[\frac{2a\sqrt{2\Delta}\left(x_0\theta_{x0} + y_0\theta_{y0}\right)}{2\Delta\left(x_0^2 + y_0^2\right) - a^2\left(\theta_{x0}^2 + \theta_{y0}^2\right)} + \pi\right] \qquad \text{(Q1.3)}$$

Otherwise we have to reverse the definitions. Inserting (Q1.2) into the $r_{\min}$ of (Q1.1), after rearrangements, we get the simple condition for meridional rays as

Any x_0, θ_{x0}, y_0, θ_{y0} satisfying $x_0\theta_{y0}=y_0\theta_{x0}$ (Q1.4)

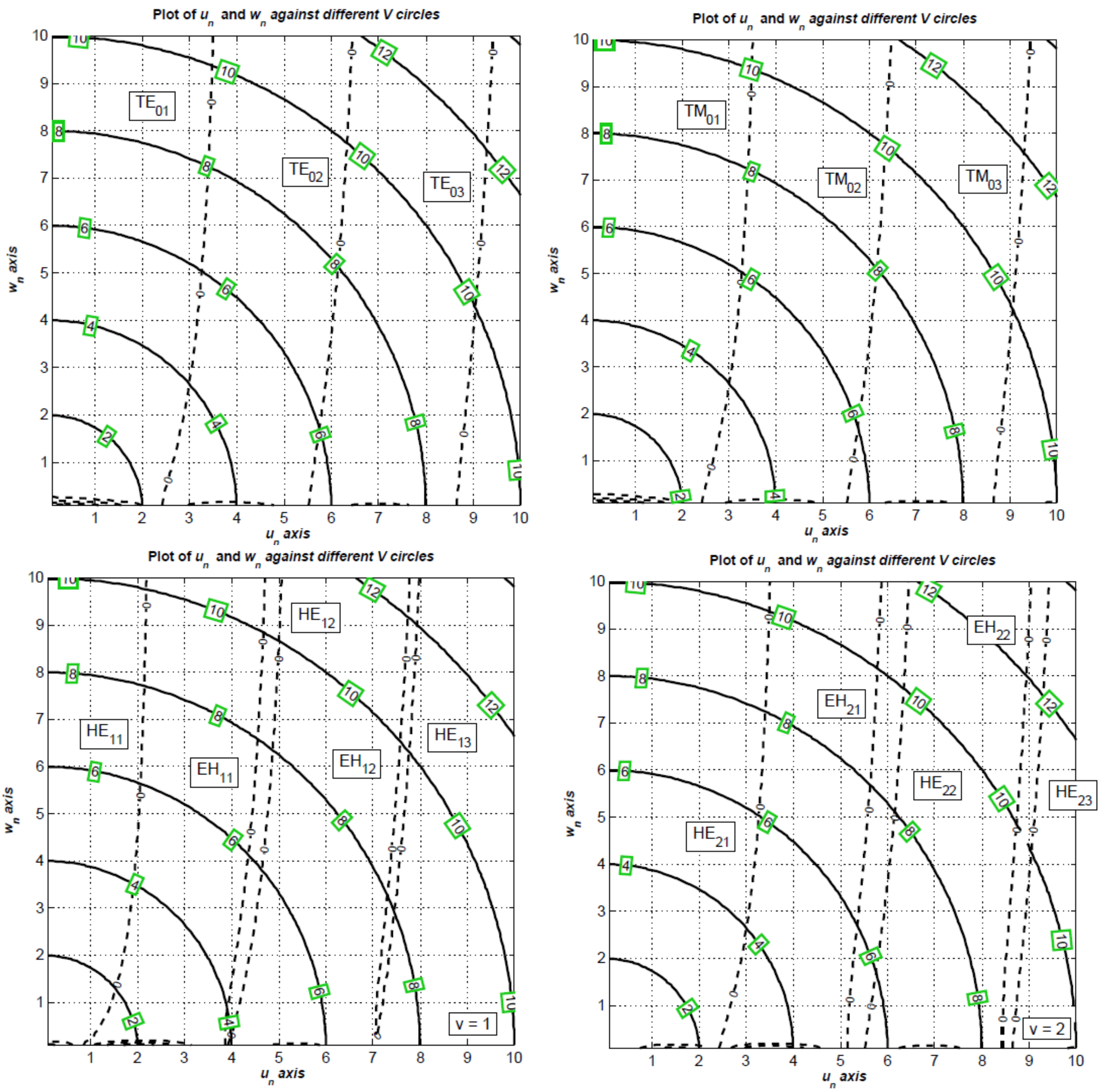
It is not equally possible to arrive a simple expression for $r_{\max} \le a$. Therefore it is better to utilize the m code supplied in Propagationin_GIFibres_Exp2 and decide whether the given ray is propagating or refracting. For the rays in A. , B., and C., taking into account the numeric values of the fibre parameter, we find

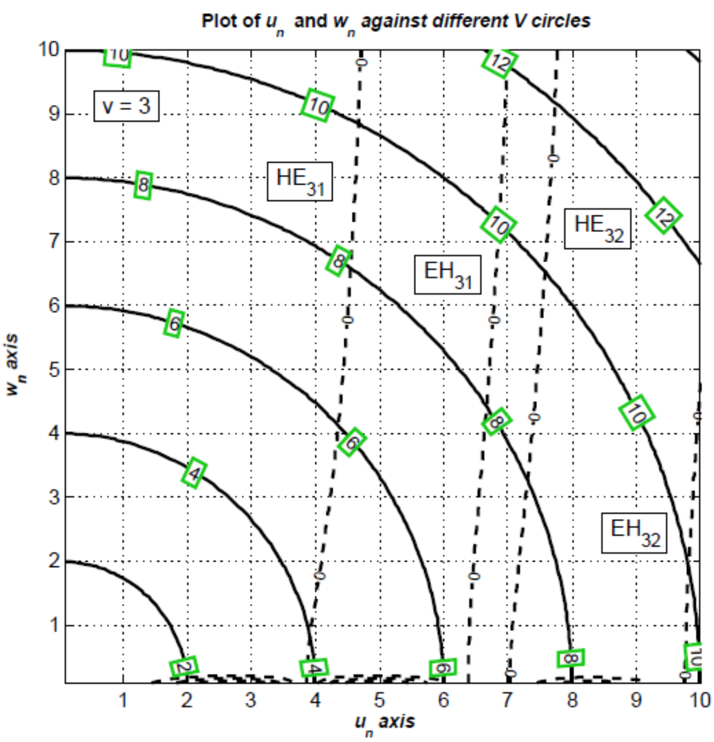
A. $x_0 = 1.2\ \mu\text{m}, y_0 = 1.2\ \mu\text{m},\ \theta_{x0}=0.1,\ \theta_{y0} = 0.2$: Refracting skew with $r_{\min} = 0.53\ \mu\text{m}, r_{\max} = 27.34\mu\text{m}, z_{p\max} = 184.5\ \mu\text{m}$

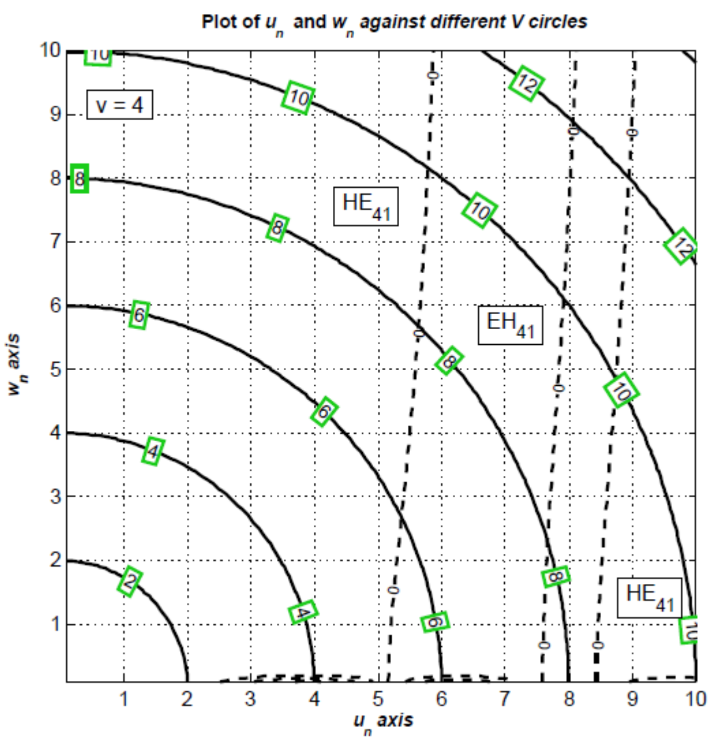
B. $x_0 = 5\ \mu\text{m}, y_0 = 7\ \mu\text{m},\ \theta_{x0}=0.4,\ \theta_{y0} = 0.3$: Refracting skew with $r_{\min} = 2.58\mu\text{m}, r_{\max} = 61.58\mu\text{m}, z_{p\max} = 175.4\ \mu\text{m}$

C. $x_0 = 25\ \mu\text{m}, y_0 = 0\ \mu\text{m},\ \theta_{x0}=0.1,\ \theta_{y0} = 0.05$: Since $x_0 = 25\ \mu\text{m} > a$ is outside the core region, such a ray cannot be accounted for

2. (35 Points) By using the graphs given below, find the propagating modes and the propagation constant (β) values of these modes in a fibre which has a core radius of $a = 5\ \mu\text{m}$, refractive index difference of $\Delta = 0.0351$, core refractive index of $n_1 = 1.49$ and operating wavelength of $\lambda = 1.55\ \mu\text{m}$. Explain the ways of making this fibre a single mode fibre. For such a single mode fibre, calculate the percentage of power propagating in the cladding.







Solution : From the given numerical values,

$$2\Delta = \frac{n_1^2 - n_2^2}{n_1^2} \quad , \quad V = ak\left(n_1^2 - n_2^2\right)^{0.5} = akn_1\left(2\Delta\right)^{0.5} \simeq 8$$

From the given graphs of TE_{0m} , TM_{0m} , $\mathrm{HE}_{\upsilon m}$ and $\mathrm{EH}_{\upsilon m}$ for $\upsilon = 1,\ 2,\ 3,\ 4$, we can calculate the propagation modes looking at the given graphs by taking those modes enclosed by the circle $V = 8$ and reading off w_n and u_n. Here for convenience, u_n is used.. Firstly k_1 and k_2 are calculated

$$k_1 = n_1 k = 6.04\text{x}10^6 \quad , \quad n_2 = \left(n_1^2 - 2n_1^2\Delta\right)^{0.5} = 1.4367 \quad , \quad k_2 = n_2 k = 5.8241\text{x}10^6$$

A. For TE_{01} $\beta = \left(\frac{w_n^2}{a^2} + k_2^2\right)^{0.5} = \left(k_1^2 - \frac{u_n^2}{a^2}\right)^{0.5}$

   Inserting the value of $u_n = 3.4$ read off from the TE_{0m} graph, we get $\beta = 6.0016\text{x}10^6$.

B. For TE_{02} reading from the same graph, $u_n = 6.1$, again using the same expression for β, we get $\beta = 5.9155\text{x}10^6$. Other higher order TE_{0m} modes are outside the $V = 8$ circle, thus we move on to TM_{0m} graph.

C. For TM_{01} by reading off from TM_{0m} graph $u_n = 3.45$, and using $\beta = \left(k_1^2 - \frac{u_n^2}{a^2}\right)^{0.5}$, we get

   $\beta = 6.0004\text{x}10^6$.

D. For TM_{02} by reading off from TM_{0m} graph $u_n = 6.15$, and using $\beta = \left(k_1^2 - \frac{u_n^2}{a^2}\right)^{0.5}$, we get $\beta = 5.9134\text{x}10^6$. Similar to the case of TE_{0m} modes, the other TM_{0m} are outside the $V = 8$ circle, therefore are refracting modes as far as this fibre is concerned.

E. For $HE_{\upsilon m}$ and $EH_{\upsilon m}$ with $\upsilon = 1$, we read the following values u_n from the relevant graph

For HE_{11} mode $u_n = 2.15$, for EH_{11} mode $u_n = 4.5$, for HE_{12} mode $u_n = 4.8$, for EH_{12} mode $u_n = 7.3$, for HE_{13} mode $u_n = 7.45$.

Using the formulation of $\beta = \left(k_1^2 - \frac{u_n^2}{a^2}\right)^{0.5}$, we find

For HE_{11} mode $\beta = 6.0246\text{x}10^6$, for EH_{11} mode $\beta = 5.9725\text{x}10^6$, for HE_{12} mode $\beta = 5.9638\text{x}10^6$, for EH_{12} mode $\beta = 5.8609\text{x}10^6$, for HE_{13} mode $\beta = 5.8533\text{x}10^6$. The higher order modes go to cut off.

F. For $HE_{\upsilon m}$ and $EH_{\upsilon m}$ with $\upsilon = 2$, we read the following values u_n from the relevant graph

For HE_{21} mode $u_n = 3.4$, for EH_{21} mode $u_n = 5.5$, for HE_{22} mode $u_n = 6.2$.

Hence for these modes, corresponding β values are

For HE_{21} mode $\beta = 6.0016\text{x}10^6$, for EH_{21} mode $\beta = 5.9390\text{x}10^6$, for HE_{22} mode $\beta = 5.9113\text{x}10^6$.

G. For $HE_{\upsilon m}$ and $EH_{\upsilon m}$ with $\upsilon = 3$, we read the following values u_n from the relevant graph

For HE_{31} mode $u_n = 4.5$, for EH_{31} mode $u_n = 6.7$, for HE_{32} mode $u_n = 7.3$.

Hence for these modes, corresponding β values are

For HE_{31} mode $\beta = 5.9725\text{x}10^6$, for EH_{31} mode $\beta = 5.8894\text{x}10^6$, for HE_{32} mode $\beta = 5.8609\text{x}10^6$.

H. For $HE_{\upsilon m}$ and $EH_{\upsilon m}$ with $\upsilon = 4$, we read the following values u_n from the relevant graph

For HE_{41} mode $u_n = 5.5$, for EH_{41} mode $u_n = 6.7$

Hence for these modes, corresponding β values are

For HE_{41} mode $\beta = 5.9390\text{x}10^6$, for EH_{41} mode $\beta = 5.8894\text{x}10^6$

Although this completes the given graphs, it is worth pointing out that also include within the $V = 8$ circle are two further modes labeled HE_{51} and HE_{61}.

To convert the given fibre so that it becomes a single mode fibre, we can adjust one or more of the followings to reduce V below 2.4

a) Lower core radius
b) Operate at higher wavelengths
c) Make the core cladding refractive index difference smaller

By adopting c) and setting $V \leq 2.4$, we get

$2.4 \geq V = akn_1 (2\Delta)^{0.5}$, deriving Δ, we find $\Delta \leq 0.0032$

By taking the single mode fibre just defined, and setting $\Delta = 0.003$, which gives $V = 2.34$, then by using Eqs. 2.2.45 and 46 of Agrawal, we initially find

$$\frac{w}{a} = 0.65 + 1.619V^{-1.5} + 2.879V^{-6} = 0.65 + 1.619\text{x}(2.34)^{-1.5} + 2.879\text{x}(2.34)^{-6} = 1.1198$$

$$\Gamma = 1 - \exp\left(-\frac{2a^2}{w^2}\right) \simeq 0.8$$

Since V is close to 2.4, the field extends little into the cladding as seen from $\frac{w}{a} = 1.1198$, thus the percentage of power travelling in the core is high, i.e. $\Gamma \simeq 0.8$.

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) Propagation in a fibre is either in the form of rays or modes : Such a description is not proper. Analysing optical propagation in terms of rays offers a simpler picture, but is possible in cases where the considered physical distances is large compared with operating wavelengths. The mode theory is applicable in all situations.

b) As the numerical aperture grows, more light enters the fibre : Partially true, numerical aperture is actually a measure of how much of the light entering the fibre will be guided.

c) Attenuation in fibres is inversely proportional to distance : False, as fibre length increases, attenuation also increases.

d) We achieve the single mode condition in a fibre by eliminating TE_{01} mode : True, this also eliminates TM_{01}.

e) There is less dispersion in graded index fibres than multimode step index fibres : True, since the differences between the arrival times of different rays seem to be more equalized.