Çankaya University – ECE Department – ECE 572

Student Name :
Student Number :

Duration : 2 hours
Open book exam

Questions

1. (30 Points) A single mode fibre has $n_1 - n_2 = 0.005$. Calculate the core radius (from normalized frequency parameter *V*) if the fibre has to have a cutoff wavelength of $1\ \mu m$. Estimate the spot size w and the fraction of mode power inside the core, when this fibre is used at $\lambda = 1.3\ \mu$m. Take $n_1 = 1.45$.

Solution: For single mode operation, $V \lesssim 2.4$

$$V \simeq \frac{2\pi}{\lambda} a n_1 \sqrt{2\Delta} \quad , \quad n_1 = 1.45, \quad \Delta = \frac{n_1 - n_2}{n_1}$$

$$n_1 - n_2 = 0.005, \quad \Delta = \frac{0.005}{1.45} = 0.0034, \quad \sqrt{2\Delta} = 0.083$$

$$a = \frac{2.4 \times 1 \times 10^{-6}}{2\pi \times 1.45 \times 0.083} = 3.17\ \mu m$$

Note that if we operate above $1\mu m$, for instance

at $\lambda = 1.31\ \mu m$ then $V = \frac{2.4}{1.31} = 1.83$

Now we can apply Eq. (2.2.45) of Agrawal

$$\frac{w}{a} = 0.65 + 1.619 V^{-3/2} + 2.879 V^{-6}$$

$w = 4.37\ \mu m$

This value of w (spot size) proves that the propagating field well extends into cladding

To find the fraction of power travelling inside the core, we use Eq (2.2.46) of Agrawal, which is

$$\Gamma = \frac{P_{core}}{P_{total}} = 1 - \exp\left(-\frac{2a^2}{w^2}\right)$$

$$= 1 - \exp\left(-\frac{2 \times 3.17^2}{4.37^2}\right)$$

$$= 0.6509$$

2. (40 Points) For single mode fibre, using the formulation and the graphs on the enclosed pages, calculate the dispersion parameter D and its individual components, D_M and D_W by taking $\lambda = 1.33\ \mu\text{m}$, $\Delta = 0.001$, n_2 to be the same as n in Fig. 2.8 and $V = 2$. Check to see if your calculated figures agree with those displayed in Fig. 2.10

Solution: If $\lambda = 1.33\,\mu m$, $f = 2.255 \times 10^{14}$ Hz

$\omega = 1.417 \times 10^{15}$ rad/sec

$$n_{2g} = n_2 + \omega \frac{dn_2}{d\omega} \quad \text{or} \quad n_g = \overset{\overset{n_1 \text{ or } n_2}{\uparrow}}{n} + \omega \frac{dn}{d\omega}$$

Assume n_{2g} is the same as n_g in Fig 2.8 of Agrawal, pp. 40. From this figure we try to estimate the slope of n_g curve at $\lambda = 1.33\,\mu m$, that is

$$\frac{dn_g}{d\lambda} (\text{at } \lambda = 1.33\,\mu m) \simeq \frac{1.4618 - 1.4617}{(1.33 - 1.6) \times 10^{-6}}$$

$$= \frac{10^{-4}}{0.27 \times 10^{-6}} = 370$$

$$D_M = \frac{1}{c} \frac{dn_{2g}}{d\lambda} \simeq \frac{1}{c} \frac{dn_g}{d\lambda} = \frac{1}{3 \times 10^8} \times 370$$

$\simeq 1.23 \times 10^{-6}$ sec $\overbrace{\text{per meter}}^{\text{for } L}$ $\overbrace{\text{per meter}}^{\text{for spectral width}}$

To convert this into ps per km per nm

$$D_M = -1.23\times10^{-6}\times10^{12}\times10^{3}\times10^{-9}\ ps/(km\,nm)$$

$$= -1.23\ ps/(km\,nm)$$

$$D_W = -\frac{2\pi\Delta}{\lambda^2}\left[\frac{n_{2g}^2}{n_2\omega}\frac{V\,d^2(Vb)}{dV^2} + \frac{dn_{2g}}{d\omega}\frac{d(Vb)}{dV}\right]$$

By inserting the numeric values and using the conversion

$$\frac{d}{d\omega} = -\frac{\lambda^2}{2\pi c}\frac{d}{d\lambda}$$

$$D_W = \frac{2\pi\times10^{-3}}{(1.33\times10^{-6})^2}\left[\frac{(1.46185)^2}{1.446\times1.417\times10^{15}}\times0.2\right.$$

$$\left. - \frac{(1.33\times10^{-6})^2}{2\times\pi\times3\times10^{8}}\times370\times0.95\right]$$

$$= 3.55\times10^{9}\left[2.0859\times10^{-16} - 3.3\times10^{-19}\right]$$

$= 7.39\times10^{-7}$ sec per meter per meter ($\times10^{6}$ for conversion)

$= 0.739\ ps/(km\,nm)$

$D = D_M + D_W = -0.49$ quite comparable to numerals displayed in Fig 2.10.

3. (30 Points) Answer the following questions as **True** or **False**. For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) Dispersion in a single mode fibre is related to variation of refractive index with frequency : True, this means that if the source is not monochromatic, then from the side of the source we see that we launch power into many fibres

b) According to ray propagation equation, rays in step index fibre travel along straight lines, whereas in graded index fibres, the rays follow a sinusoidal path : True

This is due to paraxial ray equation

$\frac{d}{dz}\left(n \frac{dx}{dz}\right) \simeq \frac{\partial n}{\partial x}$, if n has no coordinate dependence →

c) Dispersion in a graded index fibres is greater than step index fibres : False

since, less modes are excited in graded index fibres

d) Fibre modes have a radial distribution in the form of Bessel functions : True

These are in the form of Bessel functions

$$E_z = \begin{cases} A\, J_m(p r) \exp(jm\phi) \exp(j\beta z) & r \le a \\ C\, K_m(p r) \exp(jm\phi) \exp(j\beta z) & r \geq a \end{cases}$$

e) Single mode fibres have a *V* parameter whose values is less than 2 : False

To obtain single mode operation, it is sufficient to place V below the first root of $J_0(\ )$ which produces a $V < 2.405$.

then $\frac{d^2x}{dz^2} = 0$ and solution to this is

$x(z) = A_x z$, thus a straight line

Similarly for the y component

description, such broadening was attributed to different paths followed by different rays. In the modal description it is related to the different mode indices (or group velocities) associated with different modes. The main advantage of single-mode fibers is that intermodal dispersion is absent simply because the energy of the injected pulse is transported by a single mode. However, pulse broadening does not disappear altogether. The group velocity associated with the fundamental mode is frequency dependent because of chromatic dispersion. As a result, different spectral components of the pulse travel at slightly different group velocities, a phenomenon referred to as *group-velocity dispersion* (GVD), *intramodal dispersion*, or simply *fiber dispersion*. Intramodal dispersion has two contributions, material dispersion and waveguide dispersion. We consider both of them and discuss how GVD limits the performance of lightwave systems employing single-mode fibers.

2.3.1 Group-Velocity Dispersion

Consider a single-mode fiber of length L. A specific spectral component at the frequency ω would arrive at the output end of the fiber after a time delay $T = L/v_g$, where v_g is the *group velocity*, defined as [22]

$$v_g = (d\beta/d\omega)^{-1}. \tag{2.3.1}$$

By using $\beta = \bar{n}k_0 = \bar{n}\omega/c$ in Eq. (2.3.1), one can show that $v_g = c/\bar{n}_g$, where $\bar{n}_g$ is the *group index* given by

$$\bar{n}_g = \bar{n} + \omega(d\bar{n}/d\omega). \tag{2.3.2}$$

The frequency dependence of the group velocity leads to pulse broadening simply because different spectral components of the pulse disperse during propagation and do not arrive simultaneously at the fiber output. If $\Delta\omega$ is the spectral width of the pulse, the extent of pulse broadening for a fiber of length L is governed by

$$\Delta T = \frac{dT}{d\omega}\Delta\omega = \frac{d}{d\omega}\left(\frac{L}{v_g}\right)\Delta\omega = L\frac{d^2\beta}{d\omega^2}\Delta\omega = L\beta_2\Delta\omega, \tag{2.3.3}$$

where Eq. (2.3.1) was used. The parameter $\beta_2 = d^2\beta/d\omega^2$ is known as the GVD parameter. It determines how much an optical pulse would broaden on propagation inside the fiber.

In some optical communication systems, the frequency spread $\Delta\omega$ is determined by the range of wavelengths $\Delta\lambda$ emitted by the optical source. It is customary to use $\Delta\lambda$ in place of $\Delta\omega$. By using $\omega = 2\pi c/\lambda$ and $\Delta\omega = (-2\pi c/\lambda^2)\Delta\lambda$, Eq. (2.3.3) can be written as

$$\Delta T = \frac{d}{d\lambda}\left(\frac{L}{v_g}\right)\Delta\lambda = DL\Delta\lambda, \tag{2.3.4}$$

where

$$D = \frac{d}{d\lambda}\left(\frac{1}{v_g}\right) = -\frac{2\pi c}{\lambda^2}\beta_2. \tag{2.3.5}$$

D is called the *dispersion parameter* and is expressed in units of ps/(km-nm).

The effect of dispersion on the bit rate B can be estimated by using the criterion $B\Delta T < 1$ in a manner similar to that used in Section 2.1. By using ΔT from Eq. (2.3.4) this condition becomes

$$BL|D|\Delta\lambda < 1. \tag{2.3.6}$$

Equation (2.3.6) provides an order-of-magnitude estimate of the BL product offered by single-mode fibers. The wavelength dependence of D is studied in the next two subsections. For standard silica fibers, D is relatively small in the wavelength region near 1.3 μm [$D \sim 1$ ps/(km-nm)]. For a semiconductor laser, the spectral width $\Delta\lambda$ is 2–4 nm even when the laser operates in several longitudinal modes. The BL product of such lightwave systems can exceed 100 (Gb/s)-km. Indeed, 1.3-μm telecommunication systems typically operate at a bit rate of 2 Gb/s with a repeater spacing of 40–50 km. The BL product of single-mode fibers can exceed 1 (Tb/s)-km when single-mode semiconductor lasers (see Section 3.3) are used to reduce $\Delta\lambda$ below 1 nm.

The dispersion parameter D can vary considerably when the operating wavelength is shifted from 1.3 μm. The wavelength dependence of D is governed by the frequency dependence of the mode index $\bar{n}$. From Eq. (2.3.5), D can be written as

$$D = -\frac{2\pi c}{\lambda^2}\frac{d}{d\omega}\left(\frac{1}{v_g}\right) = -\frac{2\pi}{\lambda^2}\left(2\frac{d\bar{n}}{d\omega} + \omega\frac{d^2\bar{n}}{d\omega^2}\right), \tag{2.3.7}$$

where Eq. (2.3.2) was used. If we substitute $\bar{n}$ from Eq. (2.2.39) and use Eq. (2.2.35), D can be written as the sum of two terms,

$$D = D_M + D_W, \tag{2.3.8}$$

where the *material dispersion* D_M and the *waveguide dispersion* D_W are given by

$$D_M = -\frac{2\pi}{\lambda^2}\frac{dn_{2g}}{d\omega} = \frac{1}{c}\frac{dn_{2g}}{d\lambda}, \tag{2.3.9}$$

$$D_W = -\frac{2\pi\Delta}{\lambda^2}\left[\frac{n_{2g}^2}{n_2\omega}\frac{Vd^2(Vb)}{dV^2} + \frac{dn_{2g}}{d\omega}\frac{d(Vb)}{dV}\right]. \tag{2.3.10}$$

Here n_{2g} is the group index of the cladding material and the parameters V and b are given by Eqs. (2.2.35) and (2.2.36), respectively. In obtaining Eqs. (2.3.8)–(2.3.10) the parameter Δ was assumed to be frequency independent. A third term known as differential material dispersion should be added to Eq. (2.3.8) when $d\Delta/d\omega \neq 0$. Its contribution is, however, negligible in practice.

2.3.2 Material Dispersion

Material dispersion occurs because the refractive index of silica, the material used for fiber fabrication, changes with the optical frequency ω. On a fundamental level, the origin of material dispersion is related to the characteristic resonance frequencies at which the material absorbs the electromagnetic radiation. Far from the medium resonances, the refractive index $n(\omega)$ is well approximated by the *Sellmeier equation* [36]

$$n^2(\omega) = 1 + \sum_{j=1}^{M}\frac{B_j\omega_j^2}{\omega_j^2 - \omega^2}, \tag{2.3.11}$$

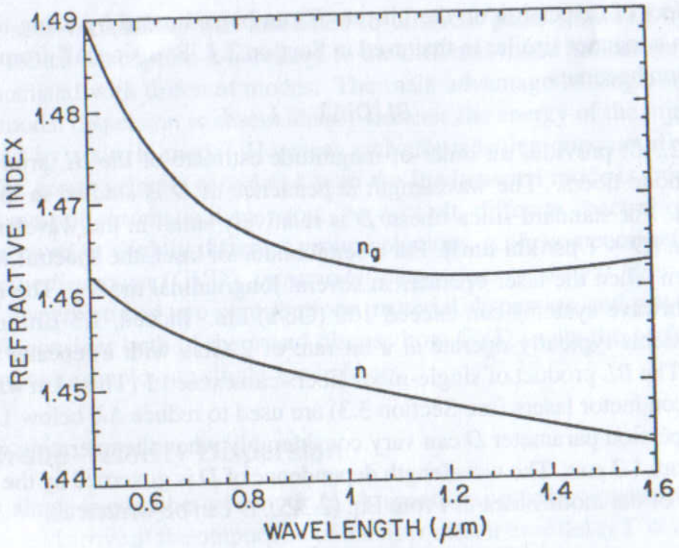

Figure 2.8: Variation of refractive index n and group index n_g with wavelength for fused silica.

where ω_j is the resonance frequency and B_j is the oscillator strength. Here n stands for n_1 or n_2, depending on whether the dispersive properties of the core or the cladding are considered. The sum in Eq. (2.3.11) extends over all material resonances that contribute in the frequency range of interest. In the case of optical fibers, the parameters B_j and ω_j are obtained empirically by fitting the measured dispersion curves to Eq. (2.3.11) with $M = 3$. They depend on the amount of dopants and have been tabulated for several kinds of fibers [12]. For pure silica these parameters are found to be $B_1 = 0.6961663$, $B_2 = 0.4079426$, $B_3 = 0.8974794$, $\lambda_1 = 0.0684043\ \mu$m, $\lambda_2 = 0.1162414\ \mu$m, and $\lambda_3 = 9.896161\ \mu$m, where $\lambda_j = 2\pi c/\omega_j$ with $j = 1$–3 [36]. The group index $n_g = n + \omega(dn/d\omega)$ can be obtained by using these parameter values.

Figure 2.8 shows the wavelength dependence of n and n_g in the range 0.5–1.6 μm for fused silica. Material dispersion D_M is related to the slope of n_g by the relation $D_M = c^{-1}(dn_g/d\lambda)$ [Eq. (2.3.9)]. It turns out that $dn_g/d\lambda = 0$ at $\lambda = 1.276\ \mu$m. This wavelength is referred to as the *zero-dispersion wavelength* λ_{ZD}, since $D_M = 0$ at $\lambda = \lambda_{ZD}$. The dispersion parameter D_M is negative below λ_{ZD} and becomes positive above that. In the wavelength range 1.25–1.66 μm it can be approximated by an empirical relation

$$D_M \approx 122(1 - \lambda_{ZD}/\lambda). \tag{2.3.12}$$

It should be stressed that $\lambda_{ZD} = 1.276\ \mu$m only for pure silica. It can vary in the range 1.27–1.29 μm for optical fibers whose core and cladding are doped to vary the refractive index. The zero-dispersion wavelength of optical fibers also depends on the core radius a and the index step Δ through the waveguide contribution to the total dispersion.

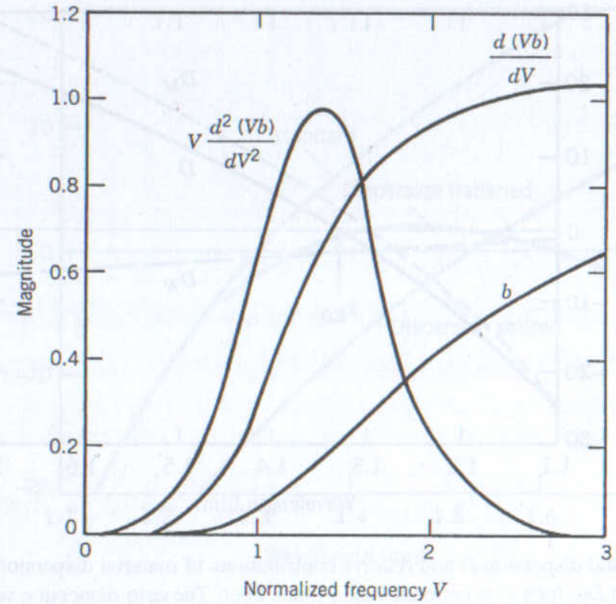

Figure 2.9: Variation of b and its derivatives $d(Vb)/dV$ and $V[d^2(Vb)/dV^2]$ with the V parameter. (After Ref. [33]; ©1971 OSA; reprinted with permission.)

2.3.3 Waveguide Dispersion

The contribution of waveguide dispersion D_W to the dispersion parameter D is given by Eq. (2.3.10) and depends on the V parameter of the fiber. Figure 2.9 shows how $d(Vb)/dV$ and $Vd^2(Vb)/dV^2$ change with V. Since both derivatives are positive, D_W is negative in the entire wavelength range 0–1.6 μm. On the other hand, D_M is negative for wavelengths below λ_{ZD} and becomes positive above that. Figure 2.10 shows D_M, D_W, and their sum $D = D_M + D_W$, for a typical single-mode fiber. The main effect of waveguide dispersion is to shift λ_{ZD} by an amount 30–40 nm so that the total dispersion is zero near 1.31 μm. It also reduces D from its material value D_M in the wavelength range 1.3–1.6 μm that is of interest for optical communication systems. Typical values of D are in the range 15–18 ps/(km-nm) near 1.55 μm. This wavelength region is of considerable interest for lightwave systems, since, as discussed in Section 2.5, the fiber loss is minimum near 1.55 μm. High values of D limit the performance of 1.55-μm lightwave systems.

Since the waveguide contribution D_W depends on fiber parameters such as the core radius a and the index difference Δ, it is possible to design the fiber such that λ_{ZD} is shifted into the vicinity of 1.55 μm [37], [38]. Such fibers are called *dispersion-shifted* fibers. It is also possible to tailor the waveguide contribution such that the total dispersion D is relatively small over a wide wavelength range extending from 1.3 to 1.6 μm [39]–[41]. Such fibers are called *dispersion-flattened* fibers. Figure 2.11 shows typical examples of the wavelength dependence of D for standard (conventional), dispersion-shifted, and dispersion-flattened fibers. The design of dispersion-

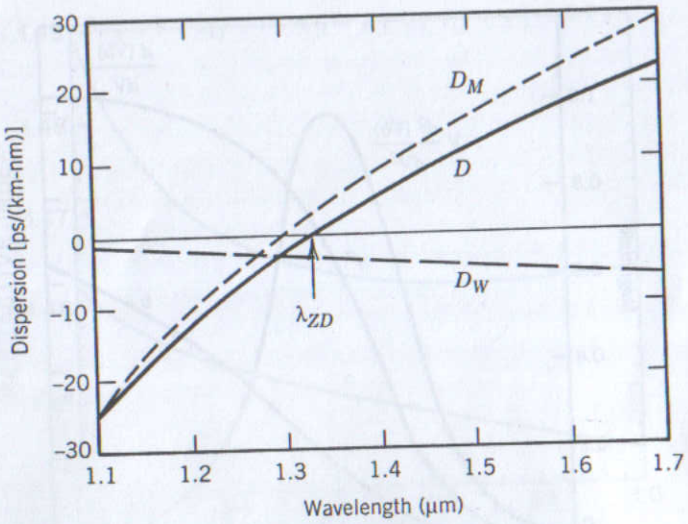

Figure 2.10: Total dispersion D and relative contributions of material dispersion D_M and waveguide dispersion D_W for a conventional single-mode fiber. The zero-dispersion wavelength shifts to a higher value because of the waveguide contribution.

modified fibers involves the use of multiple cladding layers and a tailoring of the refractive-index profile [37]–[43]. Waveguide dispersion can be used to produce *dispersion-decreasing* fibers in which GVD decreases along the fiber length because of axial variations in the core radius. In another kind of fibers, known as the *dispersion-compensating* fibers, GVD is made normal and has a relatively large magnitude. Table 2.1 lists the dispersion characteristics of several commercially available fibers.

2.3.4 Higher-Order Dispersion

It appears from Eq. (2.3.6) that the *BL* product of a single-mode fiber can be increased indefinitely by operating at the zero-dispersion wavelength λ_{ZD} where $D = 0$. The dispersive effects, however, do not disappear completely at $\lambda = \lambda_{ZD}$. Optical pulses still experience broadening because of higher-order dispersive effects. This feature can be understood by noting that D cannot be made zero at all wavelengths contained within the pulse spectrum centered at λ_{ZD}. Clearly, the wavelength dependence of D will play a role in pulse broadening. Higher-order dispersive effects are governed by the *dispersion slope* $S = dD/d\lambda$. The parameter S is also called a *differential-dispersion* parameter. By using Eq. (2.3.5) it can be written as

$$S = (2\pi c/\lambda^2)^2\beta_3 + (4\pi c/\lambda^3)\beta_2, \tag{2.3.13}$$

where $\beta_3 = d\beta_2/d\omega \equiv d^3\beta/d\omega^3$ is the third-order dispersion parameter. At $\lambda = \lambda_{ZD}$, $\beta_2 = 0$, and S is proportional to β_3.

The numerical value of the dispersion slope S plays an important role in the design of modern WDM systems. Since $S > 0$ for most fibers, different channels have slightly

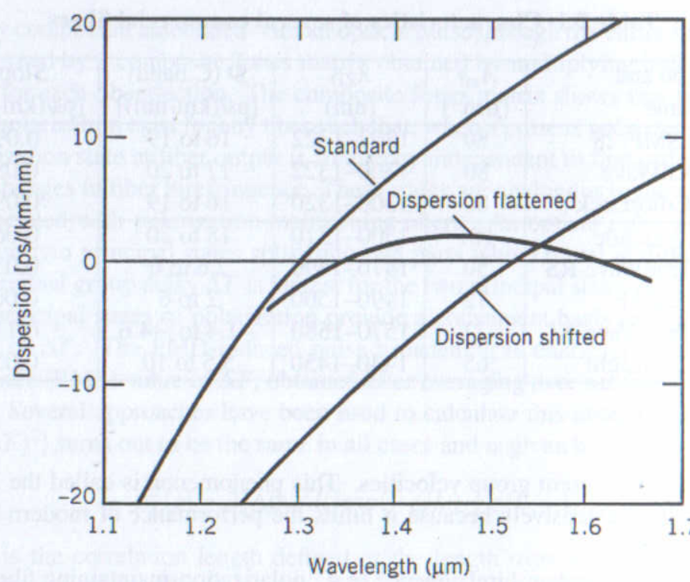

Figure 2.11: Typical wavelength dependence of the dispersion parameter D for standard, dispersion-shifted, and dispersion-flattened fibers.

different GVD values. This feature makes it difficult to compensate dispersion for all channels simultaneously. To solve this problem, new kind of fibers have been developed for which S is either small (reduced-slope fibers) or negative (reverse-dispersion fibers). Table 2.1 lists the values of dispersion slopes for several commercially available fibers.

It may appear from Eq. (2.3.6) that the limiting bit rate of a channel operating at $\lambda = \lambda_{ZD}$ will be infinitely large. However, this is not the case since S or β_3 becomes the limiting factor in that case. We can estimate the limiting bit rate by noting that for a source of spectral width $\Delta\lambda$, the effective value of dispersion parameter becomes $D = S\Delta\lambda$. The limiting bit rate–distance product can now be obtained by using Eq. (2.3.6) with this value of D. The resulting condition becomes

$$BL|S|(\Delta\lambda)^2 < 1. \tag{2.3.14}$$

For a multimode semiconductor laser with $\Delta\lambda = 2$ nm and a dispersion-shifted fiber with $S = 0.05$ ps/(km-nm^2) at $\lambda = 1.55$ μm, the *BL* product approaches 5 (Tb/s)-km. Further improvement is possible by using single-mode semiconductor lasers.

2.3.5 Polarization-Mode Dispersion

A potential source of pulse broadening is related to fiber birefringence. As discussed in Section 2.2.3, small departures from perfect cylindrical symmetry lead to birefringence because of different mode indices associated with the orthogonally polarized components of the fundamental fiber mode. If the input pulse excites both polarization components, it becomes broader as the two components disperse along the fiber