Çankaya University – ECE Department – ECE 474 (MT)

Student Name :
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Open book exam, Duration : 2 Hours

Questions

1. (40 Points) A parabolic (quadratic) graded index fibre with $n_1 = 1.50,\ n_2 = 1.47,\ a = 25\ \mu\text{m}$ is given. Using the Fermat principle i.e.,

$$\frac{d}{dz}\left(n\frac{dx}{dz}\right) \approx \frac{\partial n}{\partial z} \quad , \quad \frac{d}{dz}\left(n\frac{dy}{dz}\right) \approx \frac{\partial n}{\partial z}$$

and taking $x_0 = 0.3a,\ y_0 = 0.25a,\ \theta_{x0},\ \theta_{y0} = 0$ at $z = 0$, find the first $r_{\min}$, $r_{\max}$ and the corresponding z $\left(\text{i.e. } z_p\right)$. Repeat the same for $x_0 = 0.1a,\ y_0 = 032a,\ \theta_{x0} = 0.1,\ \theta_{y0} = 0.15$. Identify which case represents meridional ray, which case represents skew ray.

Repeat the above calculations, if the above fibre is a step index fibre.

Solution : According to formulation given in lectures and ECE 474 notes dated 23.03.2011 which are on course web page as well

$$r_{min}(z) = \left[x^2(z_p) + y^2(z_p)\right]^{1/2}, \text{ where}$$

$$z_p = \frac{a}{2\sqrt{2\Delta}}\, a\tan\left[\frac{2(x_0\theta_{x0} + y_0\theta_{y0})}{\frac{\sqrt{2\Delta}}{a}(x_0^2 + y_0^2) - \frac{a}{\sqrt{2\Delta}}(\theta_{x0}^2 + \theta_{y0}^2)}\right]$$

Then by taking $a\tan[\quad] + \pi$ and setting

$$z_{p(max)} = \frac{a}{2\sqrt{2\Delta}}\left\{a\tan[\quad] + \pi\right\}$$

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we can find $r_{max}(z)$ (or vice versa)

Inserting the given numeric values into above equations or into MATLAB m file Ray_tracing-GI-Exp2.m, we get the following results

A) At $x_0 = 0.3a$, $y_0 = 0.25a$ $\theta_{x0}, \theta_{y0} = 0$

we get $r_{min} = 0$, $r_{max} = 9.7628 \times 10^{-6}$ m

or $r_{max} = 0.3905a$ this corresponds to the r_0 position at entry into fibre since $r_{max} = (x_0^2 + y_0^2)^{1/2}$, this is also confirmed by above formulation. Additionally since $\theta_{x0} = \theta_{y0} = 0$, this is a meridional ray

B) At $x_0 = 0.1a$, $y_0 = 0.32a$

$\theta_{x0} = 0.1$, $\theta_{y0} = 0.15$

$r_{min} = 2.2191 \times 10^{-6}$ m, $r_{max} = 23.94 \times 10^{-6}$ m

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or $r_{min} = 0.088\,a$, $r_{max} = 0.9576\,a$

From the plot of Ray-tracing-GI-Exp2.m

we see that this is definitely a skew ray

(It is possible to arrive at the same conlusion

from ECE 474 Notes dated 24.03.2011)

For the step index fibre, DEs become

$$\frac{d^2 x}{dz^2} = 0 \quad , \quad \frac{d^2 y}{dz^2} = 0$$

From here and using the initial conditions

at $z = 0$ $\quad x(z=0) = x_0$, $y_0(z=0) = y_0$

$$\frac{d}{dz}[x(z)]\Big|_{z=z_0} = \theta_{x0} \quad , \quad \frac{d}{dz}[y(z=0)]\Big|_{z=z_0} = \theta_{y0}$$

Then $x(z)$ and $y(z)$ will become

$$x(z) = \theta_{x0}\, z + x_0 \quad , \quad y(z) = \theta_{y0}\, z + y_0$$

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So.. for step index fibre, the situation is more simplified. To find r_{max} and r_{min}, first we write for $r(z) = [x^2(z) + y^2(z)]^{1/2}$

$$r(z) = [\theta_{x0}^2 z^2 + 2\theta_{x0} x_0 z + x_0^2$$
$$\theta_{y0}^2 z^2 + 2\theta_{y0} y_0 z + y_0^2]^{1/2}$$

Take $\frac{d}{dz} r(z)$ and set the resulting expression to zero will give

$$z_p = \frac{-\theta_{x0} x_0 - \theta_{y0} y_0}{\theta_{x0}^2 + \theta_{y0}^2}$$

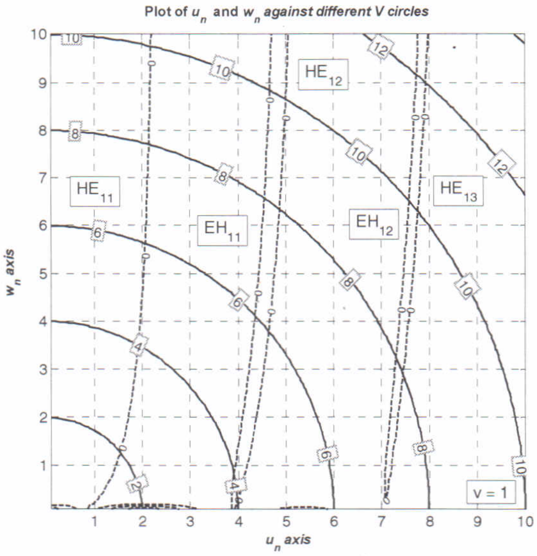
But this does not seem to produce meaningful result (to be investigated further)

For step index fibres, we know from other physical descriptions that $r_{max} = a$

2. (30 Points) The characteristic equation (CE) for EH (for upper sign) and HE (for lower sign) modes of the fibre is as given below

$$\frac{J_{\nu\pm1}(u_n)}{u_n J_\nu(u_n)} \pm \frac{K_{\nu\pm1}(w_n)}{w_n K_\nu(w_n)} = 0$$

Explain in detailed steps how we arrive at the following graph from this CE.



(zero crossings)

Solution: By the root of CE at $\nu = 1$ and setting $u_n = 0 \to 10$, $w_n = 0 \to 10$ We obtain the above graph. Here the broken lines which indicate the zeros (roots) of given CE give us the normalized propagation constant in the following manner

core radius

$$\beta_n^2 = \frac{n_1^2 V^2}{n_1^2 - n_2^2} - u_n^2 = \frac{n_2^2 V^2}{n_1^2 - n_2^2} + w_n^2 , \quad \beta_n = \beta a$$

In the graph V circles are obtained via

$$V^2 = u_n^2 + w_n^2$$

The modes are labeled as HE if we use the upper sign in CE, the indexing in $HE_{\nu m}$ is determined by the order of Bessel functions and m shows the mth root (zero crossing) of CE.

It is important to realize that given CE is applicable to step index (multimode) and single mode fibres.

To avoid singularity created by the zeros of $J_\nu(u_n)$ in denominator of CE, in MATLAB computation we use the following modified version

$$J_{\nu+1}(u_n) \pm K_{\nu\mp}(w_n) \times u_n J_\nu(u_n) / (w_n K_\nu(w_n)) = 0$$

3. (30 Points) Answer the following questions as **True** or **False**. For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer.

a) In single mode fibre, the field has no radial component in cladding : False

when $V < 2$, a substantial part of the radial component of the field propagates in the cladding.

b) NA measures how much light enters the fibre : In a way True, but we must add that NA is precisely a measure of the light that propagates by internal reflection. After all refracting rays also enter the fibre

c) The single mode condition is adjusted by setting the parameter V above 2 : An exact definition of single mode condition is

$V < 2.405$ – First root of $J_0(u_n)$

d) For meridional rays, $r_{\min} = 0$: True, since meridional rays always pass through the fibre z axis.

e) For skew rays $r_{\max} = a$ (fibre core radius) : Not necessarily, it is only in step index fibres that $r_{max} = a$ for rays that have θ_0 (entrance angle) $\neq 0$