

Çankaya University – ECE Department – ECE 474 (MT)

Student Name :
Student Number :

Date : 26.03.2012
Open book exam, Duration : 2 Hours

Questions

1. (40 Points) A parabolic (quadratic) graded index fibre has $n_1 = 1.50$, $n_2 = 1.48$, $a = 10 \mu\text{m}$. Using the Fermat principle and the related differential equations, find the range and the limits of propagating rays in this fibre in terms of initial launching conditions, i.e. x_0, y_0, θ_{x0} and θ_{y0} . In the set of parameters, x_0, y_0, θ_{x0} and θ_{y0} , identify which set of x_0, y_0, θ_{x0} and θ_{y0} will give
- a) Propagating Meridional Rays
 - b) Propagating Skew Ray
 - c) Refracting Rays

For at least two rays in category a) and b), find $r_{\min}$, $r_{\max}$ and z_p .

Solution: For classification of rays according to initial launching parameters, we use

"Notes on ray propagation in GI fibre-2703211.pdf"

To this end, we take the formulation for z_p

$$z_p = \frac{a}{2\sqrt{2\Delta}} \tan^{-1} \left[\frac{2a\sqrt{2\Delta}(x_0\theta_{x0} + y_0\theta_{y0})}{2\Delta(x_0^2 + y_0^2) - a^2(\theta_{x0}^2 + \theta_{y0}^2)} \right]$$

Assuming this z_p refers to z_p of

$r_{\min}$, then $z_{p\min} = z_p$ (of above)

and z_{pmax} which will correspond to r_{max} will be

$$z_{pmax} = \frac{a}{2\sqrt{2\Delta}} \left\{ \tan^{-1} \left[\quad \right] + \pi \right\}$$

Condition for meridional ray will be

$$r_{min} = \left[x^2(z_{pmin}) + y^2(z_{pmin}) \right]^{1/2} = 0$$

When z_{pmin} from above is substituted in the expression of r_{min} , after rearrangement, we get the simple relationship that

Any $x_0, \theta_{x0}, y_0, \theta_{y0}$ satisfying $x_0 \theta_{y0} = y_0 \theta_{x0}$

will be meridional rays (propagating and refracting)

Conditions for refraction will be

$$r_{\max} = \left[x^2(z_{p\max}) + y^2(z_{p\max}) \right]^{1/2} \pi a$$

Unfortunately in this case, it not so easy to achieve a simple relationship like in the case of $r_{\min} = 0$. One difficulty is we must be sure about $z_{p\max}$, since it is not clear which of the two z_p expressions above will give $z_{p\min}$ and $z_{p\max}$.

Here a better way is to use the MATLAB experiment m code, Ray-tracing-GL-Exp2.m.

a) Using $x_0 \theta_{y_0} = y_0 \theta_{x_0}$ for propagating Meridional Rays we may have

$$1) \quad x_0 = 0.05a, \quad \theta_{y_0} = 0.03, \quad y_0 = 0.03a, \quad \theta_{x_0} = 0.05$$

$$\text{Here } r_{\min} \approx 0, \quad r_{\max} = 3.6 \mu\text{m}, \quad z_{p\max} = 86.28 \mu\text{m}$$

$$2) \quad x_0 = 0.07a, \quad \theta_{y_0} = 0.09, \quad y_0 = 0.09a, \quad \theta_{x_0} = 0.07$$

$$\text{Here } r_{\min} \approx 0, \quad r_{\max} = 7.07 \mu\text{m}, \quad z_{p\max} = 86.28 \mu\text{m}$$

b) For propagating skew rays, we take

$$1) \quad x_0 = 0.05a, \quad y_0 = 0.03a, \quad \theta_{x_0} = 0.03, \quad \theta_{y_0} = 0.05$$

$$\text{Here } r_{\min} = 0.27 \mu\text{m}, \quad r_{\max} = 3.6 \mu\text{m}, \quad z_p = 87.38 \mu\text{m}$$

$$2) \quad x_0 = 0.07a, \quad y_0 = 0.09a, \quad \theta_{x_0} = 0.09, \quad \theta_{y_0} = 0.07$$

$$\text{Here } r_{\min} = 0.28 \mu\text{m}, \quad r_{\max} = 7.07 \mu\text{m}, \quad z_{p\max} = 86.56 \mu\text{m}$$

c) For Refracting Rays we take

$$1) x_0 = 0.15a, \theta_{y_0} = 0.09, y_0 = 0.09a, \theta_{x_0} = 0.15$$

$$\text{Here } r_{\min} \simeq 0, r_{\max} = 10.85 \mu\text{m} > a$$

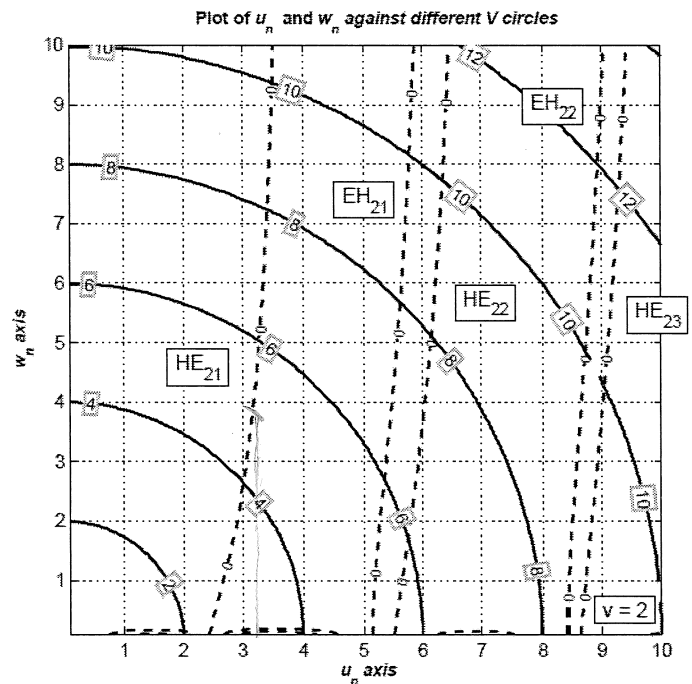
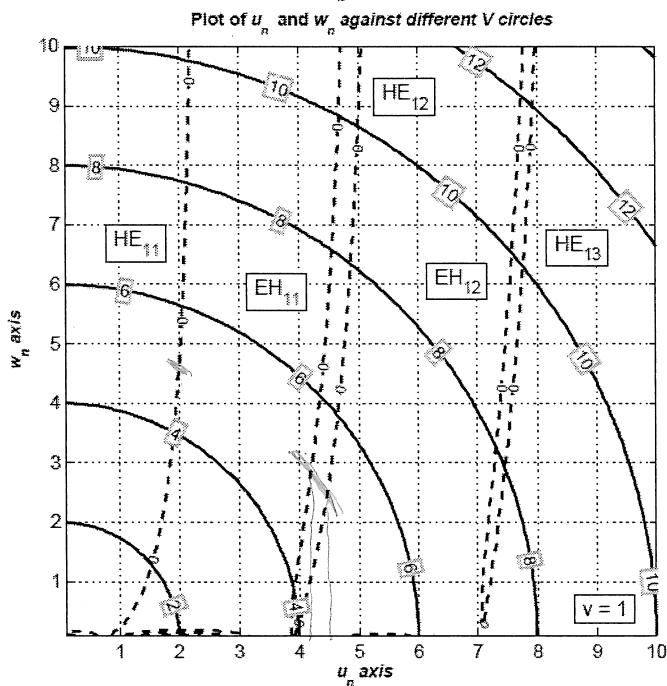
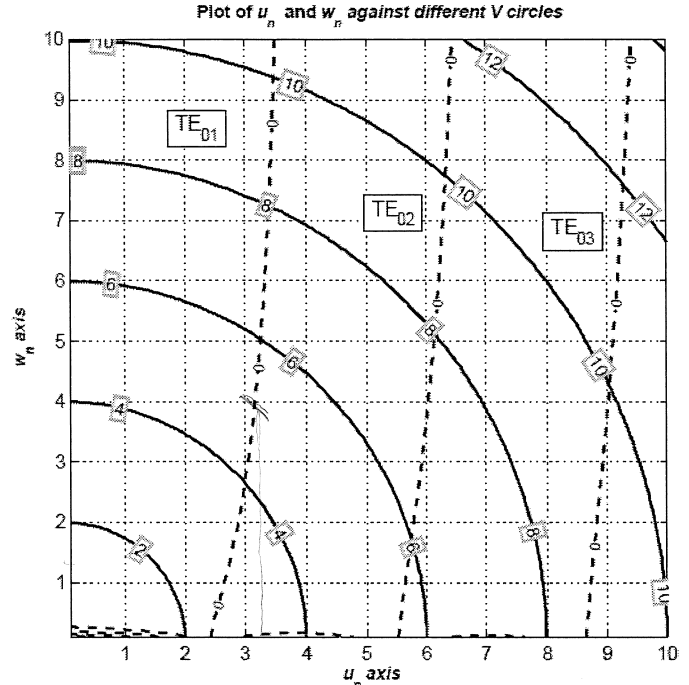
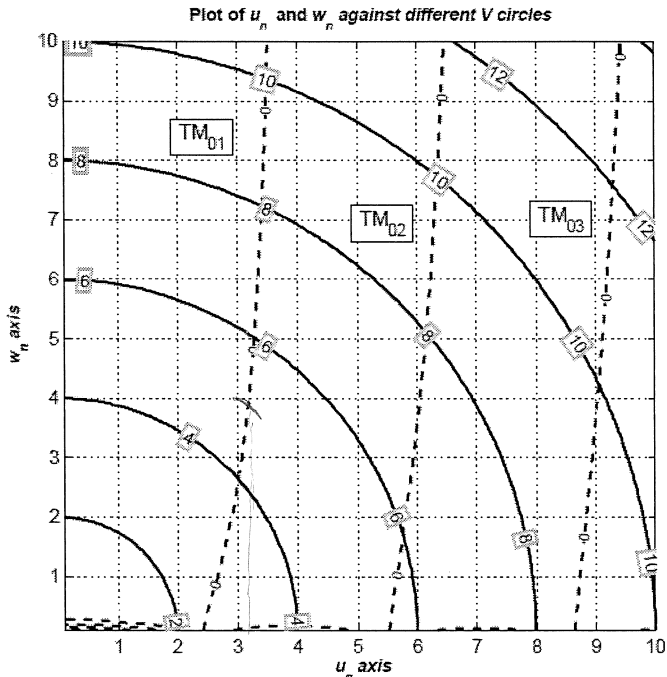
$$z_p = 86.28 \mu\text{m}$$

$$2) x_0 = 0.15a, y_0 = 0.09a, \theta_{x_0} = 0.09, \theta_{y_0} = 0.15$$

$$\text{Here } r_{\min} = 0.815 \mu\text{m}, r_{\max} = 10.82 \mu\text{m} > a$$

$$z_p = 87.38 \mu\text{m}$$

2. (30 Points) For a fibre which has core radius of $a = 4 \mu\text{m}$, refractive index difference of $\Delta = 0.0211$ and $n_1 = 1.50$, operating at $\lambda = 1.55 \mu\text{m}$, find which modes will propagate from the following graphs. Evaluate numerically the propagation constant, β of these modes. How can this fibre be made a single mode fibre?, explain and make numeric evaluations.



Solution: From the given settings

$$\frac{n_1^2 - n_2^2}{n_1^2} = 2\Delta, \quad V = ak(n_1^2 - n_2^2)^{0.5}$$

$$= ak n_1 (2\Delta)^{0.5} = 4.996382 \approx 5$$

From the given graphs of TM_{0m}, TE_{0m}, EH_{ym}, EH_{ym} (for $\nu=1$) and HE_{vm}, EH_{vm} (for $\nu=2$) we have the following propagating modes enclosed by the circle of $V=5$.

For this we find the intersections of mode root (indicated by broken line) and $V=5$ circle and heading off W_n or U_n . Here we take U_n and initially calculate

$$k_1 = n_1 k = 6.08051 \times 10^6$$

$$n_2 = (n_1^2 - 2n_1^2 \Delta)^{1/2} = 1.468$$

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$$k_2 = n_2 k = 5.951 \times 10^6$$

$$\text{For } TM_{01} \quad \beta = \left(\frac{u_n^2}{a^2} + k_2^2 \right)^{0.5} = \left(k_1^2 - \frac{u_n^2}{a^2} \right)^{0.5}$$

$$\text{Inserting } u_n = 3.15, \quad \beta = 6.0293 \times 10^6$$

$$\text{For } TE_{01}, \quad u_n = 3.2, \quad \beta = 6.0276 \times 10^6$$

$$\text{For } HE_{11}, \quad u_n = 2, \quad \beta = 6.05991 \times 10^6$$

$$\text{For } EH_{11}, \quad u_n = 4.2, \quad \beta = 5.989 \times 10^6$$

$$\text{For } HE_{12}, \quad u_n = 4.5, \quad \beta = 5.9755 \times 10^6$$

$$\text{For } HE_{21}, \quad u_n = 3.2, \quad \beta = 6.02765 \times 10^6$$

This fibre can be made a single mode

fibre by setting $V < 2.4$, which means eliminating

all modes given above except HE_{11} that has no cut-off value. As seen from the equation, $V = ak(n_1^2 - n_2^2)^{1/2}$, achieving single mode condition can be established by three means

- a) Lowering core radius, a
- b) operating at higher wavelengths
- c) Making core cladding index difference (Δ) smaller

To give a numeric example, if we keep all other parameters constant and try to achieve single mode condition by lowering a

$$V = 2.4 < ak(n_1^2 - n_2^2)^{0.5}, \text{ then}$$

$$a < 1.9214 \mu m$$

3. (30 Points) Answer the following questions as **True** or **False**. For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer.

a) In single mode fibre, the field is confined to the core : *Mainly but*

if V is below 2, then the tail of the field extends well into the cladding

b) As wavelength increases, attenuation in fibre decreases : *This is no*

such generalization, we know from the relevant graph that lowest loss window occurs at $\lambda = 1.55 \mu\text{m}$, next lowest is $\lambda = 1.31 \mu\text{m}$

c) In graded index fibres, ray path is straight lines :

False, in graded index fibre, ray path is sinusoidal as shown in the notes

d) Dispersion arises from the time delay difference between rays or modes : *True*

this is exactly the source of dispersion

e) Monochromatic optical source contains many wavelengths : *False*

Monochromatic optical source emit at a single wavelength. Polychromatic sources radiate at many wavelengths.