Optical Transmitters and Receivers

1. General

We recall the general block diagram of the optical link, and highlight the parts under study in this part of the notes.

![Block Diagram of an Optical Link](image)

Fig. 1.1 General block diagram of an fibre optical link.

Light sources that can be used on the transmitter side are light emitting diodes and laser diodes.

2. Semiconductor Optical Transmitters – Light Emitting Diodes (LEDs)

It is possible to make a pn junction to give radiation as explained below. Prior to applying any biasing to a pn junction the concentration of holes (denoted by $\Theta$) is on the p side, while that of electrons is (denoted by $\Theta$) is on the n side and inbetween there is what is called depletion region which gives rise potential barrier as shown at the bottom of Fig. 2.1. Depletion region prevents the flow of carriers and conduction from p side to n side and vice versa. We note that there are two types of carriers in semiconductors, majority and minority. For a n type material, the majority carriers are electrons, the minority ones are holes (absence of electrons), in a p type material, holes constitute majority carriers, while electrons become minority carriers. The operation of semiconductor devices is essentially based on injection and extraction of minority carriers.
Fig. 2.1 Distribution of carrier across a pn junction without an externally applied biasing.

Now applying a reverse bias as shown in Fig. 2.2, we will have a widening of depletion region, thus the flow of majority carriers will be prevented more, but with rises in the reverse potential and subsequent rise of temperature, the excessive minority carriers will be created to take part in conduction, and this is the situation for instance in illuminated photodiode.

Fig. 2.2 Reverse biasing of a pn junction.

If a forward bias is applied, the width of the depletion region becomes narrower as illustrated in Fig. 2.3. This way, the level of the potential barrier is reduced, the majority carriers from both sides cross over to the other side, increasing the minority carriers concentration on the opposite side, these excess minority carriers start recombining with the majority carriers on the side that they have crossed to. This recombination process gives rise to optical radiation and constitutes the basis of light emitting diodes.
LEDs are used as optical transmitters, at low bit rate applications, since they have wide spectral width, i.e., $\sigma_\lambda$, thus causing an appreciable amount of intramodal dispersion. There are two types of LEDs, named as “surface emitting LED” and “edge emitting LED”. In Fig. 2.4, the spectrums of both LEDs are exhibited whose emission peaks are centred around an operating wavelength of $\lambda = 1.31 \, \mu\text{m}$. As seen from there, edge emitting LED has narrower spectrum.

Fig. 2.3 Forward biasing of a pn junction.

To give an idea about the structure and the material composition, in Figs 2.5 and 2.6, we give a high radiance surface emitting coupled directly into a fibre and edge emitting double heterojunction LED.

Fig. 2.4 Typical spectrum of edge emitting and surface emitting LEDs (Fig. 4.14 of Ref [1]).
In the fabrication of LEDs, group III elements such as Al, Ga, and In, or group V elements such as P, As or Sb are used. The material composition is chosen depending on wavelength of operation. For instance, to operate in 800 to 900 nm, ternary alloy of $\text{Ga}_{1-x}\text{Al}_x\text{As}$ is used. The fraction ratio $x$ of aluminium to gallium arsenide determines the wavelength of peak emitted radiation. By choosing $x = 0.08$, we obtain the spectrum in Fig. 2.7, where the peak wavelength is $\lambda = 810$ nm and the spectral width is, $\sigma_0 = 36$ nm.
Fig. 2.7 Spectral view of an LED made of ternary alloy, $\text{Ga}_{1-x}\text{Al}_x\text{As}$ with $x = 0.08$

At longer wavelengths, we have to switch to the quaternary alloy of $\text{In}_{1-x}\text{Ga}_x\text{As}\text{P}_{1-y}$. By varying $x$ and $y$, with this quaternary alloy, it is possible optical power outputs at wavelengths between 1 $\mu$m and 1.7 $\mu$m. The peak emission wavelength in an LED is expressed as a function of the band gap energy, $E_g$ in electron volts (eV) as follows

$$\lambda (\mu\text{m}) = \frac{1.24}{E_g (\text{eV})}$$  \hspace{1cm} (2.1)

For a ternary alloy, the relationship between the band gap energy, $E_g$ and fraction ratio, $x$, when $0 \leq x \leq 0.37$ is given by

$$E_g = 1.424 + 1.266x + 0.266x^2$$  \hspace{1cm} (2.2)

**Example 2.1**: Consider a ternary alloy of $\text{Ga}_{1-x}\text{Al}_x\text{As}$, if $x = 0.07$, find the wavelength of operation.

**Solution**: From (2.2), we compute the band gap energy as

$$E_g = 1.424 + 1.266 \times 0.07 + 0.266 \times 0.07^2 = 1.51 \text{ eV}$$  \hspace{1cm} (2.3)

Then using (2.1), we have

$$\lambda (\mu\text{m}) = \frac{1.24}{E_g (\text{eV})} = \frac{1.24}{1.51} = 0.82 \mu\text{m}$$  \hspace{1cm} (2.4)

In quaternary alloys, the fraction ratio $y$ has also to be taken into account. For $0 \leq x \leq 0.47$, $E_g$ will be given in terms of $y$ as
Example 2.2: For a quaternary alloy of $\text{In}_{1-x} \text{Ga}_x \text{As}_y \text{P}_{1-y} \rightarrow \text{In}_{0.74} \text{Ga}_{0.26} \text{As}_{0.57} \text{P}_{0.43}$, find the wavelength of operation.

Solution: From the given alloy, we read $x = 0.26, y = 0.57$, then using (2.5),

\[ E_g = 1.35 + 0.72y + 0.12y^2 \]

Subsequently from (2.1), we get

\[ \lambda (\mu m) = \frac{1.24}{E_g (eV)} = \frac{1.24}{0.97} = 1.27 \mu m \]

3. Laser Diodes

Laser diodes are quite different from LEDs basically in two ways. Firstly, lasers are able to offer narrower spectral width, thus causing less intramodal dispersion. Secondly, lasers are known as coherent sources, which means that different portions of the light emitted by lasers act in harmony with each other in spatial (space) and temporal (time) domain. The property of coherence is important in some applications.

We can talk about three processes to describe the act of lasing. These are absorption, spontaneous emission and simulated emission. These are shown in Fig. 3.1.

![Principles of absorption, spontaneous emission and simulated emission](image)

Fig. 3.1 The principles of absorption, spontaneous emission and simulated emission (lasing).

According to Planck’s law, if an electron is in ground state, i.e. $E_i$ in Fig. 3.1, can be excited to an upper energy level of $E$, by supplying an (photon) external energy of $hf_{12}$, where is $h$ Planck’s constant, $f_{12}$ is the frequency of the external optical energy such that the energy of $hf_{12}$ is sufficient to raise the electron from the ground state, $E_i$ to the excited state, $E_2$. The energy level of $E_2$ for
the electron is a not a stable state, so eventually, this electron will fall from $E_2$ to $E_1$ giving out a radiated energy of $hf_{12}$. Note that this action is spontaneous, not externally controlled, therefore appears as narrowband Gaussian noise.

The downward transition of the electron from $E_2$ to $E_1$ can also be induced by the supply of external photon energy of $hf_{12}$. This way, the radiated energy of the electron is in phase with the externally supplied energy. This inphase radiation forms the basis of coherence.

We must add that the process of raising enough electrons from $E_1$ to $E_2$ is called population inversion.

The radiation in a laser diode is generated within a Fabry-Perot resonator cavity. Its mechanical construction is shown in Fig. 3.2. As seen from there, laser radiation is emitted in the longitudinal direction from the lasing spot. Since we are mainly interested in what goes on in the resonator cavity, we show in Fig. 3.3 a simplified side view together with the mirrors at the cavity ends. To describe how this lasing takes place, we express the electric field in the longitudinal direction as follows

$$E(z, t) = I(z) \exp \left[ j(\omega t - \beta z) \right]$$  \hspace{1cm} (3.1)

$I(z)$ is the optical intensity, $\omega$ is the optical radial frequency and $\beta$ is the propagation constant.

Fig. 3.2 Fabry-Perot resonator cavity for a laser diode (Fig. 4.18 of Ref [1]).
Population inversion must be achieved so that optical amplification and thus the lasing can start. For this to happen, gains in the cavity must overcome losses. After traversing a distance of $z$ in the cavity, the optical intensity will become

$$I(z) = I(0) \exp\left[\left(\Gamma g - \alpha\right)z\right]$$

(3.2)

Where $\Gamma$ is called the confinement factor, $g$ represents the gain, $\alpha$ is the absorption coefficient. As seen from Fig. 3.3, one round trip covers a length of $z = 2L$ and involves reflections from mirrors with reflectivity coefficients of $R_1$ and $R_2$, which are given by

$$R_1 = \left(\frac{n_1 - n_r}{n_1 + n_r}\right)^2, \quad R_2 = \left(\frac{n_2 - n_r}{n_2 + n_r}\right)^2$$

(3.3)

Where $n_1$ and $n_2$ are the refractive indices of the first and second mirrors, $n_r$ is the refractive index of the resonator cavity medium. Hence after being reflected from the mirrors, (3.2) will become

$$I(z) = I(0) R_1 R_2 \exp\left[\left(\Gamma g - \alpha\right)z\right]$$

(3.4)

For oscillations to occur, the magnitude and the phase of the returned wave must be equal at $z = 0$ and $z = 2L$, thus

$$I(z = 2L) = I(z = 0) \quad \text{for amplitude}$$

$$\exp\left(j2\beta L\right) = 1 \quad \text{for phase}$$

(3.5)

**Example 3.1**: For a GaAs laser diode, $R_1 = R_2 = 0.32$, absorption coefficient $\alpha = 10 \text{ cm}^{-1}$ and cavity length is $L = 500 \mu\text{m}$, find the minimum value for the product of confinement and gain factor, i.e., $\Gamma g$ for lasing to start.
Solution: From (3.4) and (3.5), we get

\[ I(z = 0) = I(z = 0)R_1R_2 \exp[(\Gamma g - \bar{\alpha})2L] \]

\[ \Gamma g = \bar{\alpha} + \frac{1}{2L} \ln \left( \frac{1}{R_1R_2} \right) = 33 \text{ cm}^{-1} \]  

(3.6)

The second line of (3.5) is satisfied at integer multiples of \( 2\pi \), hence

\[ \exp(j2\beta L) = \exp(j2\beta m), \quad 2\beta L = 2\pi m \]  

(3.7)

Now we know that is the wave number in a medium of certain refractive index, thus

\[ \beta = kn = \frac{2\pi n}{\lambda}, \quad m = \frac{2nL}{\lambda} = \frac{2nL}{c}f \]

\[ \lambda = \frac{2nL}{m}, \quad f = \frac{mc}{2nL} \]  

(3.8)

The implication on the second line of (3.8) is that the phase condition on the second line of (3.5) is satisfied for an infinite number of wavelengths and frequencies, since we are at liberty to let \( m \) go to infinity. This is practically not feasible of course, since the gain parameter, \( g \) is wavelength dependent in the manner of

\[ g(\lambda) = g(\lambda_0)\exp \left[ -\frac{(\lambda - \lambda_0)^2}{2\sigma^2} \right] \]  

(3.9)

where \( \lambda_0 \) indicates the central frequency of the gain spectrum and \( \sigma^2 \) is the variance (the width) of this spectrum. As seen from (3.9), gain drops on either side of the central frequency. Thus the laser output will also be shaped in the same way, as shown for a typical GaAlAs/GaAs laser diode in Fig. 3.4.
Fig. 3.4 Normalized gain spectrum GaAlAs/GaAs laser diode.

To examine the spectral wavelength or frequency spacings of the laser outputs, we use the first line of (3.8) and write for the $m$ th and $(m - 1)$ th wavelengths and the frequencies

$$m = \frac{2nL}{\lambda_m}, \quad m - 1 = \frac{2nL}{\lambda_{m-1}}$$

$$m = \frac{2nL}{c} f_m, \quad m - 1 = \frac{2nL}{c} f_{m-1}$$

(3.10)

By taking the differences in (3.10), we get

$$m - (m - 1) = \frac{2nL}{\lambda_m} - \frac{2nL}{\lambda_{m-1}}, \quad \Delta \lambda = \frac{\lambda^2}{2nL}$$

$$m - (m - 1) = \frac{2nL}{c} f_m - \frac{2nL}{c} f_{m-1}, \quad \Delta f = \frac{c}{2nL}$$

(3.11)

where we have denoted $\lambda_m - \lambda_{m-1} = \lambda^2$.

A typical appearance of wavelength spacings from a Fabry-Perot resonator is exhibited in Fig. 3.5.

Fig 3.5 Typical wavelength spacings from a Fabry-Perot resonator Fabry-Perot laser resonator.
Example 3.2: A GaAs laser has a central wavelength of $\lambda_0 = 850$ nm. Its cavity length is $L = 500 \mu$m and the refractive index in the cavity is $n_r = 3.7$. If the half power point is $\lambda - \lambda_0 = 2$ nm, find the wavelength, frequency spacings, $\Delta \lambda$, $\Delta f$ and the spectral width, $\sigma$.

Solution: By using (3.11), we get

$$\Delta \lambda = \frac{\lambda^2}{2n_r L} = \frac{(850 \text{ nm})^2}{2 \times 3.7 \times 500 \text{ nm}} = 0.195 \text{ nm}$$

$$\Delta f = \frac{c}{2n_r L} = \frac{3 \times 10^8 \text{ m/s}}{2 \times 3.7 \times 500 \text{ nm}} = 81 \text{ GHz} \quad (3.12)$$

To estimate the spectral width, we convert (3.9) and perform the calculation as shown below

$$\exp\left[-\frac{(\lambda - \lambda_0)^2}{2\sigma^2}\right] = \frac{g(\lambda)}{g(\lambda_0)} = 0.5 \quad , \quad \sigma^2 = \frac{(\lambda - \lambda_0)^2}{-2 \ln(0.5)}$$

$$\sigma^2 = \frac{(2 \text{ nm})^2}{-2 \ln(0.5)} = 2.8854 \times 10^{-18} \quad , \quad \sigma \approx 1.7 \text{ nm} \quad (3.13)$$

Exercise 3.1: A laser diode has a central wavelength of $\lambda_0 = 1550$ nm, with a spectral width of $\sigma = 8$ nm and a peak gain of $g(\lambda_0) = 30 \text{ cm}^{-1}$. The cavity length is $L = 200 \mu$m and the refractive index in the cavity is $n_r = 3.2$. Using the Gaussian spectral form, plot $g(\lambda)$ against $\lambda$. Calculate how many modes will be excited in this laser. Find the wavelength, $\Delta \lambda$ and frequency spacings, $\Delta f$ of these modes. If the modes with amplitude less than 0.1 times of the central (peak) wavelength of $\lambda_0 = 1550$ nm are to be discarded, estimate the number of remaining modes above this amplitude level.

Finally on the optical transmitters we present a graph in Fig. 3.6 exhibiting how an LED and laser diode are biased and modulated by an electrical message signal.
As seen from Fig. 3.6, LED is quite linear starting from the zero diode current. But the same cannot be said for laser diode which contains a nonlinear region at low diode currents which must be avoided to get the normal laser behaviour. Additionally laser diodes require extremely stable current sources to maintain the same wavelength of radiation. Otherwise, the spectrum broadening will be experienced.

4. Semiconductor Optical Receivers – pin Photodiode

The most commonly used photodetector is pin photodiode. It consists of p and n regions with a very lightly doped intrinsic region. When reverse biased, it gives an (electrical) output upon incident light falling on the intrinsic region. The associated circuitry and the energy band diagram explaining the principle of operation are respectively given in Fig. 4.1 and 4.2.
As seen from Figs. 4.1 and 4.2, the reverse bias creates a big potential barrier across the intrinsic region, also widening the depletion region. If the photodiode is illuminated and the energy in this incidence is larger than the band gap energy of the photodiode, then electrons can be excited from valance band into conduction band. This way, free electron hole pairs, known as photocarriers are generated. Then the high electric field present in the depletion region causes these carriers to separate and get collected across the reversed biased junction. This eventually gives rise photodetector current, $I_p$, also following through an externally connected load resistor. Since the incident photon energy is (directly) proportional to frequency. Thus in a manner similar to (2.1), the cut-off wavelength of the pin photodiode will be determined by

$$\lambda_c (\mu m) = \frac{hc}{E_g (eV)} = \frac{1.24}{E_g (eV)} \tag{4.1}$$

where $h$ is the Planck’s constant, i.e. $h = 6.625 \times 10^{-34}$ Js and $c$ is the speed of light, i.e. $c = 3 \times 10^8$ ms$^{-1}$. The cut-off wavelength $\lambda_c$, is about 1.06 $\mu$m for Si and 1.6 $\mu$m.

**Example 4.1 :** A photodiode constructed of GaAs, has a band gap energy of 1.43 eV. Find the highest wavelength it will operate at.

**Solution :** From (4.1), we have

$$\lambda_c (\mu m) = \frac{hc}{E_g (eV)} = \frac{(6.625 \times 10^{-34} \text{ Js})(3 \times 10^8 \text{ ms}^{-1})}{(1.43 \text{ eV})(1.6 \times 10^{-19} \text{ eV}^{-1})} = 869 \text{ nm} \tag{4.2}$$

The photodetector current generated under such illumination is dependent on the parameters given in (4.3), namely

$$I_p = \frac{q}{hf} P_0 [1 - \exp(-\alpha_w)][1 - R_v] \tag{4.3}$$
$q$ is the electron charge, $P_o$ is the incident optical power, $\alpha$, is the absorption coefficient of the surface of the photodiode, $w$ is the width of the depletion region, $R_f$ is the reflectivity. Based on (4.3), we can define two important parameters, one is the quantum efficiency, denoted by $\eta$, the other responsivity denoted by $R$. The related definitions are given below.

$$\eta = \frac{\text{number of electron hole pairs generated}}{\text{number of incident photons}} = \frac{I_p/q}{P_o/hf} = \frac{1 - \exp(-\alpha w)}{(1 - R_f)}$$  \hspace{0.5cm} (4.4)

**Example 4.2**: In a 100 ns pulse $6 \times 10^5$ photons fall onto a InGaAs photodiode. On average, $5.4 \times 10^6$ electron hole pairs are generated. Find the quantum efficiency of this photodiode.

**Solution**: Using (4.4), we find

$$\eta = \frac{\text{number of electron hole pairs generated}}{\text{number of incident photons}} = \frac{5.4 \times 10^6}{6 \times 10^5} = 0.9$$ \hspace{0.5cm} (4.5)

For a practical photodetector diode, the quantum efficiency will range between 0.3 and 0.95. As seen from (4.3), one way to achieve high quantum efficiency is to increase the width of depletion layer, $w$. However larger depletion layer widths also cause slower response times, so a compromise has to be made.

Another parameter of characterization for the photodiode is its responsivity, which is given by

$$R = \frac{I_p}{P_o} = \frac{\eta q}{hf}$$ \hspace{0.5cm} (4.6)

From (4.6), it is clear that responsivity $R$ measures the photocurrent generated per unit optical power. In general both the quantum efficiency and the responsivity are wavelength dependent. In this context, Fig. 4.3 exhibits the quantum efficiency and the responsivity of Si, InGaAs and Ge.
Fig. 4.3 Wavelength dependence of quantum efficiency and responsivity (Fig. 6.4 of Ref. [1]).

**Example 4.3:** Photons of energy $1.53 \times 10^{-19}$ J are incident on a photodiode, which has a responsivity of $0.65$ AW$^{-1}$. If the optical power is $10 \mu$W, find the photocurrent generated.

**Solution:** Using (4.6), we find that

$$I_p = RP_e = (0.65 \text{ AW}^{-1})(10 \mu\text{W}) = 6.5 \mu\text{A} \quad (4.7)$$

**Example 4.4:** As seen from Fig. 4.3, the quantum efficiency and the responsivity are wavelength dependent, for instance, for an InGaAs photodiode, using (4.6), the responsivity at any wavelength and quantum efficiency is

$$R = \frac{\eta q}{h f} = \frac{\eta q \lambda}{h c} = \frac{\eta (1.6 \times 10^{-19} \text{ C}) \lambda}{(6.625 \times 10^{-34} \text{ Js})(3 \times 10^8 \text{ ms}^{-1})} = 8.0556 \times 10^5 \eta \lambda \quad (4.8)$$

So at the wavelength of $\lambda = 1300$ nm, the quantum efficiency is around $60\%$, then we will have a responsivity of

$$R = 0.6 \times (8.0556 \times 10^5 \text{ AW}^{-1} \text{m}^{-1})(1.3 \times 10^{-6} \text{ m}) = 0.63 \text{ AW}^{-1} \quad (4.9)$$

At the higher wavelength of $\lambda = 1600$ nm, on the other hand, the quantum is approximately the same, then at $\lambda = 1600$ nm, the responsivity will become
In a photo diode made of a ternary alloy, In\textsubscript{0.53}Ga\textsubscript{0.47}As, the band gap energy, $E_g$ is 0.7 eV, using (2.1), the upper limit of the diode’s response would be

$$\lambda = \frac{1.24}{E_g (\text{eV})} = \frac{1.24}{0.73} = 1.7 \text{ \mu m}$$

(4.11)

5. Semiconductor Optical Receivers – Avalanche Photodiode

The operation principle of avalanche photodiode is based on multiplying the generated photocurrent (carriers) before the input circuitry of the receiver. This way, only the signal is amplified, not the noise. A schematic diagram is shown in Fig. 5.1. As seen from this figure, the multiplication is achieved by high electric field in what is called the avalanche region.

![Electric field diagram](image)

Fig. 5.1 The variation of the electric field for an avalanche photodiode (Fig. 6.5 of Ref. [1]).

The multiplication factor in an avalanche photodiode is defined as

$$M = \frac{I_M}{I_p}$$

(5.1)

where $I_M$ is the photodetector current after multiplication, whereas $I_p$ is the current prior to multiplication.

**Example 5.1**: A silicon avalanche photodiode has a quantum efficiency of $\eta = 0.65$ at a wavelength of 900 nm. An incident optical power of $P_0 = 0.5 \mu W$ produces a multiplied photo current of $I_M = 10 \mu A$, find the multiplication factor, $M$. 

$$R = 0.7 \times \left(8.0556 \times 10^2 \text{ AW}^{-1}\text{m}^{-1}\right) \times \left(1.6 \times 10^{-6} \text{ m}\right) = 0.77 \text{ AW}^{-1}$$

(4.10)
**Solution:** Using (4.6), we initially find $I_p$ as

$$I_p = R P_o = \frac{\eta q}{\hbar f} P_o = \frac{\eta q \lambda}{\hbar c} P_o = \frac{(0.65) \left(1.6 \times 10^{-19} \text{ C} \right) \left(9 \times 10^{-7} \text{ m} \right)}{(6.625 \times 10^{-34} \text{ Js}) \left(3 \times 10^8 \text{ ms}^{-1} \right)} \times 5 \times 10^{-7} \text{ W} = 0.235 \mu A \ (5.2)$$

Now, from (5.1), we get

$$M = \frac{I_m}{I_p} = \frac{10 \mu A}{0.235 \mu A} = 43 \quad \ (5.3)$$

These notes are based on

4) My own ECE 474 Lecture Notes.